International Journal of Heat and Mass Transfer 123 (2018) 979–987

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A numerical treatment for partial slip flow and heat transfer of non-Newtonian Reiner-Rivlin fluid due to rotating disk

Maria Tabassum, M. Mustafa *

School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan

A R T I C L E I N F O

Article history:
Received 1 November 2017
Received in revised form 7 February 2018
Accepted 12 March 2018

Keywords:
Reiner-Rivlin fluid
Slip condition
Rotating disk
Temperature jump
Boundary layer

A B S T R A C T

In this article, partial slip flow of Reiner-Rivlin fluid induced by a rough rotating disk is modeled. Heat transfer is also addressed by assuming more general temperature jump condition. The constitutive relations in Reiner-Rivlin fluid lead to a coupled and strongly non-linear differential system. A convenient numerical treatment is invoked to solve the resulting similarity equations for broad ranges of non-Newtonian fluid parameter and slip coefficients. Our main interest is to predict the behaviors of fluid elasticity and wall slip coefficients on the von-Kármán flow problem. Expressions of wall skin friction and surface heat transfer are calculated and deliberated for broad parameter values. Different from radial and axial velocities, tangential velocity appears to increase as Reiner-Rivlin fluid parameter (K) increases. Reduction in surface drag coefficient, which is vital in some applications, can be accomplished by increasing the parameter K. Volumetric flow rate is also inversely proportional to the parameter K. However, heat transfer rate diminishes when parameter K enlarges. We also conclude that larger torque would be required to keep steady rotation of the disk for higher values of wall slip coefficients.

1. Introduction

Fluid flow induced by a rotating disk has been a compelling research topic since it is relevant in a number of technical applications involving electrochemical systems, deposition of coatings on surfaces, rotor-stator system, atmospheric and oceanic circulations, viscometer and various others. The seminal contribution in this area was made by von-Kármán [1]. He assumed that a disk of large radius rotates with constant angular velocity in a still fluid. Near the disk, the lack of centrifugal effect causes the fluid to leave the disk in the radial direction. Fluid above the disk replaces this fluid through a downward spiraling motion often referred as disk free pumping effect. von-Kármán's work has led to many subsequent research activities concerning rotating disk flows. For example, an accurate asymptotic solution to the von- Kármán's problem was presented by Cochran [2]. Heat transfer analysis for von-Kármán problem was made by Millsaps and Pohlhausen [3] for a variety of Prandtl numbers. Ackroyd [4] investigated suction phenomenon for fluid flow due to permeable rotating disk and developed series solution in terms of exponentially decaying functions. Bachelor [5] showed that von-Kármán problem is a limiting case of the family of flows resulting when both disk and ambient fluid rotate with different angular velocities about the same axis. Another special case in which fluid at infinity is in state of rotation while the disk is stationary was first addressed by Bödewadt [6]. A comprehensive review of literature concerning rotating disk induced flows was presented by Zandbergen and Dijkstra [7]. Ariel [8] obtained series approximations for swirling flow of viscoelastic fluid bounded by a rotating disk. Miclavcic and Wang [9] modeled slip flow by a rough rotating disk and provided accurate numerical computations for broad range of slip coefficients. Chawla et al. [10] extended the von- Kármán's problem for the case in which fluid at infinity also rotates about the same axis as that of the disk. Turkyilmazoglu [11] explored stagnation-point flow due to stretchable rotating disk in the existence of transverse magnetic field. Recent attempts in this area can be found through [12–16] and refs. there in.

Despite the fact that almost all industrial fluids are non-Newtonian, von- Kármán's analysis for non-Newtonian fluids has been scarcely attempted. Andersson et al. [17] modeled the power-law fluid flow caused by a rotating disk. They proposed a reliable numerical approach that yielded accurate numerical calculations even for highly shear-thickening fluids. Attia [18] addressed ion-slip effects on swirling flow of Reiner-Rivlin fluid caused by a rotating disk. Later, Sahoo [19] considered the similar problem by incorporating Joule heating, partial slip and heat transfer effects. Ahmadpour and Sadeghy [20] explored the von-Kármán problem

* Corresponding author.
E-mail address: meraj_mm@hotmail.com (M. Mustafa).

https://doi.org/10.1016/j.ijheatmasstransfer.2018.03.040
0017-9310/

for Bingham fluids. Their analysis revealed that yield stress in Bingham fluid contributes to a growth in minimum torque needed to maintain steady disk rotation. Griffiths [21] considered the rotating flow of generalized Newtonian fluid obeying Carreau viscosity model which is practically applicable for vanishing small and infinitely large shear rates. A mathematical model for flow of power-law fluid induced by a rotating disk having variable thickness was developed by Xun et al. [22]. Very recently, Doh and Muthtamilselvan [23] described thermophoretic diffusion in steady micropolar fluid flow caused by a rotating disk.

Fluids such as large molecular weight polymers that do not follow the Newtonian constitutive relation are frequently encountered in chemical and plastic industries. Paints, clay, nylon, slurries, detergents, blood, lubricants, colloids, melted chocolate, egg whites, mayonnaise, gelatin etc. exhibit non-Newtonian behavior. The stress inside viscoelastic fluids does not vanish instantly upon the removal of stress due to sustained stress by intermolecular structure. This unique characteristic is termed as memory effect. The non-Newtonian fluid model given by Reiner [24] and Rivlin [25] can adequately predict flow behaviors of many geological and biological materials as well as many food products and polymers. Furthermore, liquid slip has implication in several applications including liquid coatings, lubrication, MEMS and bio-MEMS applications. Moreover, it is difficult to satisfy no-slip at the disk when the disk is rough or its surface contains impurities. In this situation, the no-slip condition can be approximated with the partial slip conditions. Despite the aforementioned applications, the treatment of partial slip conditions in non-Newtonian fluid flows can be rarely found in the literature. A few recent examples can be found through [26–33].

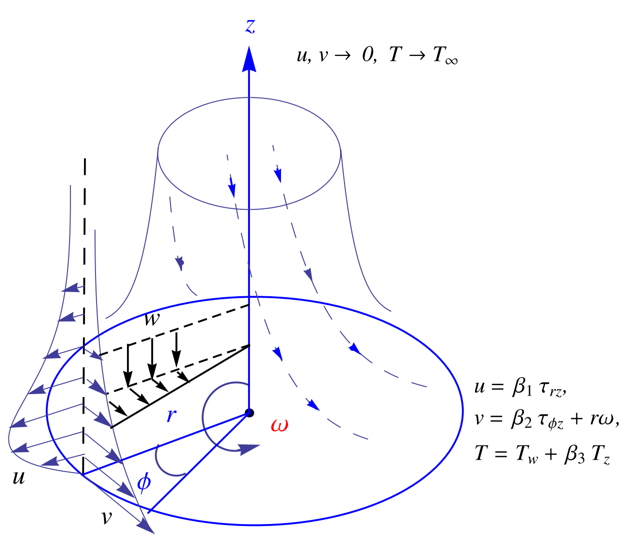

Fig. 1. Physical sketch of the problem.

The objective of this research is to explore slip flow of non-Newtonian Reiner-Rivlin fluid by a rough rotating disk. It is important to state that equations embodying the momentum transport in Reiner-Rivlin fluids considered in [18,19], and in some other articles are unfortunately no error free, as described in detail later. Hence, correct formulation for the momentum equations is just presented in this paper. Heat transfer problem is formulated through temperature jump condition. In present analysis, the consideration of wall slip leads to a non-linear boundary condition. Different from [18] or [19], we adopt shooting approach with fifth-order Runge-Kutta method to provide accurate numerical calculations for full range of wall slip coefficients. The remaining article is arranged in the following way. Section 2 covers the problem formulation. The details concerning the applied numerical scheme are given in Section 3. Section 4 contains the physical description to the role of pertinent parameters of the problem. Concluding remarks are presented in Section 5.

2. Problem formulation

We consider the steady flow of an incompressible Reiner-Rivlin fluid occupying semi-infinite region above an infinite disk coinciding with the plane $z = 0$. The disk is in a state of rigid body rotation about the vertical axis with constant angular velocity ω that sets up a swirling flow in the neighboring fluid layers. A physical sketch of the problem is shown in Fig. 1. Let u, v and w be the components of velocity along the directions of increasing r, φ and z respectively. Because of the axial symmetry, the velocity components are assumed to be independent of the azimuthal coordinate φ. Partial slip conditions are implemented considering that characteristic scale of protuberance is small compared to the boundary layer thickness. Let T_w be the constant temperature at the disk and T_∞ is the fluid temperature high above the surface. We make use of the temperature jump condition is the current analysis. Reiner [24] and Rivlin [25] have developed the following constitutive relation:

$$\tau_{ij} = -p\delta_{ij} + \mu e_{ij} + \mu_c e_{ik} e_{kj}; \quad e_{jj} = 0, \tag{1}$$

in which τ_{ij} denotes the stress tensor, p represents pressure, μ is the co-efficient of viscosity, μ_c represents the cross-viscosity coefficient, δ_{ij} is the Kronecker symbol and $e_{ij} = (\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)$ is the deformation rate tensor. Relevant equations describing fluid motion and heat transfer over a rotating disk are given below:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r}\right) = \frac{\partial \tau_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\tau_{rr} - \tau_{\varphi\varphi}}{r}, \tag{3}$$

$$\rho\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\varphi}) + \frac{\partial \tau_{z\varphi}}{\partial z} + \frac{\tau_{r\phi} - \tau_{\phi r}}{r}, \tag{4}$$

Table 1
Comparison of present findings with those of Turkyilmazoglu and Senel [12] in uniform roughness case ($\lambda_1 = \lambda_2$) with $K = 0$.

λ_1	λ_2	$F''(0)$[a]	$G'(0)$[a]	$F(\infty)$[a]	$F''(0)$[b]	$G'(0)$[b]	$F(\infty)$[b]
0	0	0.5102326	−0.6159220	0.442237	0.5102332	−0.6159219	0.442228
1	1	0.1279236	−0.3949276	0.394738	0.1279241	−0.3949280	0.394713
5	5	0.0185885	−0.1433882	0.291882	0.0185883	−0.1433879	0.291842
10	10	0.0068125	−0.0810300	0.243792	0.0068125	−0.0810301	0.243797
20	20	0.0023615	−0.0437884	0.199987	0.0023615	−0.0437883	0.199904
40	40	0.0007901	−0.0229953	0.162113	0.0007899	−0.0229951	0.160963

[a] Denote results obtained by Turkyilmazoglu and Senel [12].
[b] Denote results obtained by present authors.