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Constructal branching design for fluid flow and heat transfer

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ABSTRACT

Our focus is space-filling networks to deliver and distribute flows. Here we report an analytical analysis of optimal branching networks of tubes for both fluid flow and heat transfer. This attempt results in the structural features of these networks, mainly on relationships between the size of the parent and daughter tubes at bifurcations, and the branching angles of the bifurcations. The process of construction of these networks is described for both laminar and turbulent flow, Newtonian and power-law fluids, and constant and pulsatile flows. The extended design rules obtained in this study are compared with the optimal branching rules available in the literature. Structural features of bifurcating tubes under different flow regimes are also analyzed.

1. Introduction

Natural and engineering flow systems rely on tree-shaped networks for an efficient delivery of fluid and mass, and heat transfer function. The optimal branching of tree flow networks has been the subject of numerous studies owing to its importance in understanding the natural and manmade systems. Cardiovascular and respiratory systems rely on tree network of vessels and airways to perform their function. Besides bringing blood close to cells so that the exchange of nutrients and wastes, cardiovascular system serves an additional purpose of heat exchanger [1]. Engineering networks for fluid flow and heat transfer function thrive in several systems such as cooling systems [2–6].

We learned from Nature that the arrangement of vessels in branching systems is of functional significance. Studies [7,8] show that when the parent tube branches into daughter tubes, the rule is that the cube of the diameter of the parental tube equals the sum of the cubes of the diameters of the daughter tubes. This rule minimizes the work required to enable fluid flow, and is usually termed as Hess-Murray law. It is worthwhile to mention that, the application of other optimization principles results in the same law [6–16]. Murray [9] also applied this rule to derive the equations for the optimum branching angles of vascular system. By considering the minimum work required to drive a laminar flow, Murray found that the symmetric bifurcation angle is about 75°.

Hess-Murray's rule was derived using biological considerations, and it is of great value in the description of vessels of vascular tree, as well as the airways, but it can be also applied to non-living networks [2–6,17]. Following the derivation of this law by Murray and other authors [8–15,18], it is clear that the Hess-Murray rule is valid only on the assumption of a Hagen-Poiseuille flow, of a Newtonian fluid under a steady constant pressure gradient, through a tube of rigid and non-permeable walls. Throughout the years, this rule has been extended to include, namely, non-Newtonian fluids of power law type [10,11], and turbulent flow in rough tubes [12]. For power law fluids, the optimal ratio for diameter increases with the fluid behavior index. In the fully-rough and fully-turbulent regimes, the ratio of parent and daughter tubes' diameters is equal to a homothetic factor of $2^{-3/7}$. Other studies extended Hess-Murray's rule to account for tubes with permeable walls [13,14], and another researches predicted the optimum ratio between consecutive lengths of bifurcating tubes [12,15]. In fact, there are several examples that may serve to confirm Hess-Murray's rule and extensions (see for example [6,16–20]).

Filling a space with an efficient branching network is not irrelevant, when the flow system must reliably distribute fluid, materials or other, at each level. The efficiency in material and energy use, rendered in the minimization of work, power or resistances, shapes the structure of these networks. While authors have explored the

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relationships between the size and position of the parent and daughter tubes at the bifurcation, under isothermal or quasi-isothermal systems, branching is also useful for the design of engineering systems, such as cooling systems [2,6]. At each level, the design of parent and daughter tubes may be influenced by heat effects, and motivate an investigation.

This paper is focused on space-filling networks to optimal delivery of fluid and heat. Here, we present a theoretical foundation for incorporating heat effect in the fluid flow framework to understand the connection between individual tubes in dichotomous systems. As a result, the formulation presented here may contribute to establish design rules to improve engineered tree flow networks, but also to better understand the diversity of network morphologies observed in some natural systems.

2. Theory

Consider a flow of a fluid in a straight cylindrical tube subject to a constant wall heat flux. In addition to head loss due to friction in the tube, heat increases the temperature resulting in an acceleration of the fluid. The pressure drop through the tube is

$$\frac{dP}{dx} = \frac{f_{DW}}{D}\left(\frac{\rho_f}{2}u^2\right) + u^2\frac{d\rho_f}{dx} \tag{1}$$

where P is the pressure, D is the diameter, ρ_f is the fluid density, f_{DW} is the Darcy-Weisbach friction factor and u is the fluid velocity. Notice that the right-hand terms of Eq. (1) represent the friction pressure drop and the acceleration pressure drop, respectively. The temperature-induced density gradient is given by

$$\frac{d\rho_f}{dx} = a_T\rho_f\frac{dT}{dx} \tag{2}$$

and the temperature gradient and the heat flow are related by

$$Q = \frac{\rho_f u D A_{sc}}{4}(c_p - u^2 a_T)\frac{dT}{dx} \tag{3}$$

where A_{sc} is the surface area of the side of the tube, and a_T is much less than 1 for liquid and gases ($\sim -T^{-1}$ for nearly ideal gases) [21]. Therefore, it may be assumed that $c_p\text{-}u^2a_T \sim c_p$, and substituting Eqs. (2) and (3) into Eq. (1) results in

$$\frac{dP}{dx} = \frac{16(f_{DW}+f_{ac})}{\pi^2D^5}\left(\frac{\rho_f}{2}\phi^2\right) \tag{4}$$

with

$$f_{ac} = \frac{2\pi a_T DQ}{\rho_f L c_p\phi} \tag{5}$$

Here ϕ is the is the volumetric flow rate, and the Darcy-Weisbach friction factor may be calculated by [22,23]

$$f_{DW} = \frac{c_{DW}}{Re_D^n} \tag{6}$$

with

$$c_{DW} = 64,\ \ n = 1\ \ Re_D \leqslant 2100\ \text{(laminar flow)} \tag{7.1}$$

$$c_{DW} = 0.316,\ \ n = 1/4\ \ 2100 < Re_D \leqslant 2\times10^4\ \text{(fully − smooth tubes)} \tag{7.2}$$

$$c_{DW} = \left(\frac{1}{1.14 - 2\log(\varepsilon)}\right)^2,\ \ n = 0\ 0.00004 < \varepsilon < 0.05,\ 6\times10^3 < Re_D < 10^7\text{(fully − rough tubes)} \tag{7.3}$$

where Re_D is the Reynolds number, and ε is the relative roughness of the tube.

Consider a flow system composed by a tube that splits into two tubes (Fig. 1). The analogy between fluid flow and current is intuitive and can be directly applied. The general case of resistance to current is the impedance because covers the cases of phase shift. In fact, pure resistance (current in-phase with the applied potential) is a measure of impedance. For a tube that branches off into two tubes (Fig. 1), analogous to resistors, the impedance of daughter tubes are in parallel, but the equivalent impedance of daughter tubes (given by the reciprocal of the sum of the reciprocals of the individual impedances) and the impedance of parent tube are in series.

Let the impedance at the junction of parent and daughter tubes small when compared with the impedance of parent and daughter tubes. For fluid flow, this means that the svelteness factor defined by the ratio of the external to the internal length scales is higher than the square root of 10 [24]. Therefore, the impedance Z of a single tube is given by

$$\frac{\Delta P}{\phi} = Z\phi^{1-n} \tag{8}$$

and the total impedance of system formed by parent and daughter tubes becomes

$$Z_{total}\phi^{1-n} = Z_p\phi^{1-n} + \frac{Z_{d1}\phi_{d1}^{1-n}Z_{d2}\phi_{d2}^{1-n}}{Z_{d1}\phi_{d1}^{1-n} + Z_{d2}\phi_{d2}^{1-n}} \tag{9}$$

where the subscripts p and d mean parent and daughters tubes, respectively.

For symmetrical branching tubes $Z_{d1} = Z_{d2}$, $\phi_{d1} = \phi_{d2}$ and $\phi = 2\phi_d$, and Eq. (9) reduces to

$$Z_{total} = Z_p + \frac{Z_d}{2}\left(\frac{\phi_d}{\phi}\right)^{1-n} = Z_p + \frac{Z_d}{2^{2-n}} \tag{10}$$

where n is 1 for laminar flow (Eq. (7.1)), 1/4 for fully-smooth turbulent flow (Eq. (7.2)), and 0 for fully-rough turbulent flow (Eq. (7.3)).

2.1. Optimal branching size of tubes under laminar flow and with heat flow

2.1.1. Constant Newtonian fluid flow

For a Newtonian flow in a tube, the Reynolds number is given by $4\rho_f\phi/\pi D\mu$, where μ is the fluid viscosity. The impedance Z of a single tube can be obtained substituting Eqs. (5), (6) and (7.1) into Eq. (4), which results in

$$\frac{\Delta P}{\phi} = Z = \frac{16}{\pi}\frac{L}{D^4}\left(8\mu + \frac{a_TQ}{c_pL}\right) \tag{11}$$

For the branching system of Fig. 1, combining Eqs. (10) and (11) one obtains

$$Z_{total} = \frac{16}{\pi}\frac{L_p}{D_p^4}\left(8\mu + \frac{a_TQ}{c_pL_p}\right) + \frac{1}{2}\frac{16}{\pi}\frac{L_d}{D_d^4}\left(8\mu + \frac{a_TQ}{c_pL_d}\right) \tag{12}$$

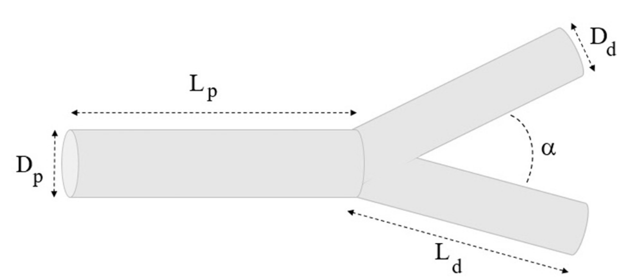

Fig. 1. Schematic representation of a single tube that splits into two tubes.