International Journal of Heat and Mass Transfer 118 (2018) 66–74

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Two-phase natural convection dusty nanofluid flow

Sadia Siddiqa [a,*], Naheed Begum [b], M.A. Hossain [c], Rama Subba Reddy Gorla [d], Abdullah A.A.A. Al-Rashed [e]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Kamra Road, Attock, Pakistan
[b] Institute of Applied Mathematics (LSIII), TU Dortmund, Vogelpothsweg 87, D-44227 Dortmund, Germany
[c] Department of Mathematics, University of Dhaka, Bangladesh
[d] Department of Mechanical & Civil Engineering, Purdue University Northwest, Westville, IN 46391, USA
[e] Department of Automotive and Marine Engineering Technology, College of Technological Studies, The Public Authority for Applied Education and Training, Kuwait

ARTICLE INFO

Article history:
Received 12 August 2017
Received in revised form 15 October 2017
Accepted 16 October 2017

Keywords:
Natural convection
Nanofluid
Two-phase
Dusty fluid
Wavy surface

ABSTRACT

An analysis is performed to study the two-phase natural convection flow of nanofluid along a vertical wavy surface. The model includes equations expressing conservation of total mass, momentum and thermal energy for two-phase nanofluid. Primitive variable formulations (PVF) are used to transform the dimensionless boundary layer equations into a convenient coordinate system and the resulting equations are integrated numerically via implicit finite difference iterative scheme. The effect of controlling parameters on the dimensionless quantities such as skin friction coefficient, rate of heat transfer and rate of mass transfer is explored. It is concluded from the present analysis, that the diffusivity ratio parameter, N_A and particle-density increment number, N_B have pronounced influence on the reduction of heat transfer rate.

1. Introduction

The analysis of nanofluids have received a notable attention because of their tremendous spectrum of applications including sterilization of medical suspensions, nanomaterial processing, automotive coolants, microbial fuel cell technology, polymer coating, intelligent building design, microfluid delivery devices and aerospace tribology. The term nanofluid, first coined by Choi [1], refers to a liquid containing a dispersion of submicron solid particles (nanoparticles) having higher thermal conductivity in a base fluid. It is noteworthy that these nanoparticles are taken ultrarefine (*i.e.*, length of order 1–50 nm), so that nanofluids appear to behave more like a single-phase fluid than a solid-liquid suspension. The nanoparticles used in nanofluids are usually made of chemically stable metals, oxides, carbides, nitrides, or non-metals, and the base fluid is generally a conductive fluid, such as water, ethylene glycol (or other coolants), oil (and other lubricants), polymer solutions, bio-fluids and other common fluids. Because of the enhanced heat transfer characteristics and useful applications, numerous investigations have been made on nanofluids under various physical circumstances. In this context, book by Das et al. [2] and review papers in [3–8] presents comprehensive discussion of published work on convective heat transfer in nanofluids.

The investigations on flow of fluids with suspended particles have attracted the attention of numerous researchers due to their practical applications in various problems of atmospheric, engineering and physiological fields [9]. Farbar and Morley [10] were the first to analyze the gas-particulate suspension on experimental grounds. After that, Marble [12] studied the problem of dynamics of a gas containing small solid particles and developed the equations for gas-particle flow systems. Singleton [13] was the first to study the boundary layer analysis for dusty fluid and later on, the dynamics of two-phase flow was investigated by numerous authors under different physical circumstances [14–20].

It is noteworthy to mention here that irregular surfaces, say, vertical or horizontal wavy surfaces have been considered vastly in the literature [21–27]. Through these analysis, it has been reported that such surfaces serves practically in engineering applications (for instance in solar collectors, grain storage containers, industrial heat exchangers and condensers in refrigerators). Although the single-phase flow of nanofluids over flat and/or wavy geometries have been considered extensively in the literature, but no work has been reported on the problem of effect of solid inert (dust) particles on heat and mass transfer of two-phase nanofluid flows along irregular surfaces. The goal of this work is to develop numerical computational techniques which can be applied to two-phase dusty nanofluid flows along vertical wavy surface. The mathematical model considered in present problem is a dusty fluid model proposed by Saffman [11], which handles the discrete phase

* Corresponding author.
E-mail addresses: saadiasiddiqa@gmail.com, sadia_siddiqa@ciit-attock.edu.pk (S. Siddiqa), naheed.awan6@gmail.com (N. Begum).

https://doi.org/10.1016/j.ijheatmasstransfer.2017.10.067
0017-9310/© 2017 Elsevier Ltd. All rights reserved.

(particles) and the continuous phase (fluid) as two continua occupying the same space. In our work, the nanoparticles are assumed to move due to such phenomena as Brownian motion and thermophoresis and are carried by the flow of the base fluid. Furthermore, water is taken as continuous base fluid that contains dust particles in it. Our purpose is to investigate the combined effect of surface roughness element and dust particles on the flow and heat transfer phenomena for the class of nanofluids. The Navier-Stokes and energy equations are coupled with nanoparticle volume fraction, dusty phase and amplitude of surface waviness to describe the phenomenon systematically. Taking Grashof number Gr_L to be very large, the boundary layer approximation is invoked, leading to a set of non-similar parabolic partial differential equations whose solution is obtained through implicit finite difference method. From the present analysis, we will interrogate whether the presence of dust particles in nanofluids affects the physical characteristics associated with the wavy surfaces or not? The computational data is presented graphically in the form of wall shear stress, heat transfer rate and mass transfer rate by varying several controlling parameters. In addition, streamlines, isotherms, velocity and temperature profiles are plotted to observe the flow pattern within the boundary layer.

2. Flow analysis

A two-dimensional natural convection flow of two-phase dusty nanofluid is modeled along a heated vertical wavy surface. The boundary layer analysis outlined below allows the shape of the wavy surface, $\bar{y}_w = \bar{\sigma}(\bar{x})$, to be arbitrary, but our detailed numerical work will assume that the surface exhibits sinusoidal deformations. Therefore, the shape of wavy surface profile is assumed to pursue the following pattern:

$$\bar{y}_w = \bar{\sigma}(\bar{x}) = \bar{a}\sin\left(\frac{2\pi\bar{x}}{L}\right) \tag{1}$$

where $\bar{a}$ is the amplitude of the transverse surface wave and L is the characteristic length associated with the wave. The schematic of the geometry are given in Fig. 1. In the above equations, over-bars denotes the dimensional quantities. The surface of vertical wavy plate is maintained at a constant temperature T_w, which is higher than the ambient fluid temperature, T_∞. The assumption of two-phase flow has been extensively analyzed in the past (for details see Refs. [11,19]), and the equations describing the complete

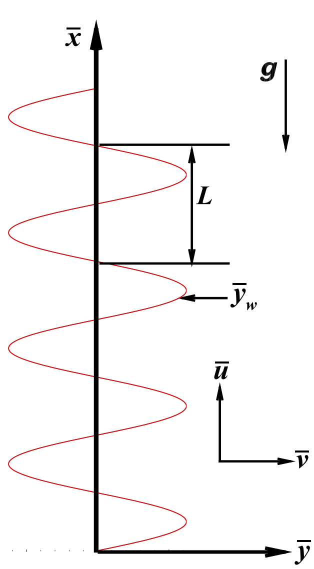

Fig. 1. Physical model.

description of the convective flow along vertical surface can be written in dimensional form as:

For fluid phase:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{2}$$

$$\rho_f\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + \mu_f\nabla^2\bar{u} + \rho_f g\beta(T - T_\infty)(1-\phi_\infty) - g(\rho_{np}-\rho_f)(\phi-\phi_\infty) + \frac{\rho_p}{\tau_m}(\bar{u}_p - \bar{u}) \tag{3}$$

$$\rho_f\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \mu_f\nabla^2\bar{v} + \frac{\rho_p}{\tau_m}(\bar{v}_p - \bar{v}) \tag{4}$$

$$\rho_f c_f\left(\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}}\right) = \kappa_f\nabla^2 T + (\rho c)_{np}\left(D_B\nabla\phi.\nabla T + \frac{D_T}{T_\infty}\nabla T.\nabla T\right) + \frac{\rho_p c_s}{\tau_T}(T_p - T) + \frac{\rho_p}{\tau_m}\{(\bar{u}_p-\bar{u})^2 + (\bar{v}_p-\bar{v})^2\} \tag{5}$$

$$\bar{u}\frac{\partial\phi}{\partial\bar{x}} + \bar{v}\frac{\partial\phi}{\partial\bar{y}} = D_B\nabla^2\phi + \frac{D_T}{T_\infty}\nabla^2 T \tag{6}$$

For particle phase:

$$\frac{\partial\bar{u}_p}{\partial\bar{x}} + \frac{\partial\bar{v}_p}{\partial\bar{y}} = 0 \tag{7}$$

$$\rho_p\left(\bar{u}_p\frac{\partial\bar{u}_p}{\partial\bar{x}} + \bar{v}_p\frac{\partial\bar{u}_p}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}_p}{\partial\bar{x}} - \frac{\rho_p}{\tau_m}(\bar{u}_p - \bar{u}) \tag{8}$$

$$\rho_p\left(\bar{u}_p\frac{\partial\bar{v}_p}{\partial\bar{x}} + \bar{v}_p\frac{\partial\bar{v}_p}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}_p}{\partial\bar{y}} - \frac{\rho_p}{\tau_m}(\bar{v}_p - \bar{v}) \tag{9}$$

$$\rho_p c_s\left(\bar{u}_p\frac{\partial T_p}{\partial\bar{x}} + \bar{v}_p\frac{\partial T_p}{\partial\bar{y}}\right) = -\frac{\rho_p c_s}{\tau_T}(T_p - T) \tag{10}$$

where $(\bar{u}, \bar{v}), T, \phi, \bar{p}, \rho_f, c_f, \beta\ \kappa_f, \mu_f$ are respectively the velocity vector in the $(\bar{x}, \bar{y})$ directions, temperature, concentration of nanoparticles, pressure, density, specific heat at constant pressure, volumetric expansion coefficient, thermal conductivity and dynamic viscosity of the suspension of nanofluid. Similarly, $(\bar{u}_p, \bar{v}_p), T_p, \bar{p}_p, \rho_p$ and c_s corresponds to the velocity vector, temperature, pressure, density and specific heat for the particle phase. In addition, g is the gravitational acceleration, τ_m (τ_T) the momentum relaxation time (thermal relaxation time) for dust particles, ϕ_w the nanoparticle volume fraction near the surface, ϕ_∞ the nanoparticle volume fraction at the outer edge of the boundary layer region, ρ_{np} the density of the nanoparticles, D_B the Brownian diffusion coefficient and D_T the thermophoretic diffusion coefficient.

The fundamental equations stated above are to be solved under the appropriate boundary conditions to determine the flow fields of the fluid and the dust particles. Therefore, the boundary conditions for the problem under considerations are:

For fluid phase:

$$\begin{aligned} &\bar{u}(\bar{x}, \bar{y}_w) = \bar{v}(\bar{x}, \bar{y}_w) = T(\bar{x}, \bar{y}_w) - T_w = \phi(\bar{x}, \bar{y}_w) - \phi_w = 0 \\ &\bar{u}(\bar{x}, \infty) = T(\bar{x}, \infty) - T_\infty = \phi(\bar{x}, \infty) - \phi_\infty = 0 \end{aligned} \tag{11}$$

For particle phase:

$$\begin{aligned} &\bar{u}_p(\bar{x}, \bar{y}_w) = \bar{v}_p(\bar{x}, \bar{y}_w) = T_p(\bar{x}, \bar{y}_w) - T_w = 0 \\ &\bar{u}_p(\bar{x}, \infty) = T_p(\bar{x}, \infty) - T_\infty = 0 \end{aligned} \tag{12}$$

In order to transform all the variables given in Eqs. (2)–(12), in uniform order of magnitude, the following continuous dimensionless variables have been employed: