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Review

Radial integration boundary element method for nonlinear heat conduction problems with temperature-dependent conductivity

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ABSTRACT

In this paper, a new and simple boundary-domain integral equation is presented to solve nonlinear heat conduction problems with temperature-dependent conductivity of materials. The boundary-domain integral equation is formulated for nonlinear heat conduction problems by using the fundamental solutions for the corresponding linear heat conduction problems, which results in the appearance of a domain integral due to the variation of the heat conductivity with temperature. The arising domain integral is converted into an equivalent boundary integral using the radial integration method (RIM) by expressing the temperature as a series of basis functions. This treatment results in a pure boundary element algorithm and requires no internal cells to evaluate the domain integral. To solve the final system of algebraic equations formed by discretizing the boundary of the problem into boundary elements, the Newton–Raphson iterative method is applied. Numerical examples are presented to demonstrate the accuracy and efficiency of the present method.

Contents

1. Introduction

The conventional boundary integral equations dealing with non-homogeneous [1,2] and non-linear [3,4] heat conduction problems include domain integrals. To evaluate these domain integrals, the computational region needs to be discretized into internal cells, which makes BEM lose its distinct advantage of only boundary discretization. To circumvent this deficiency, some methods of transforming domain integrals into equivalent boundary integrals are proposed and have been frequently used in BEM. In these methods, the dual reciprocity method (DRM) developed by Brebbia [5,6] is extensively utilized. However, DRM requires particular solutions to the basis functions, which restricts its application to the complicated problems. Recently, a new transformation method, the radial integration method (RIM), has been developed

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by Gao [7,8], which not only can transform any complicated domain integrals to the boundary in a unified way without using particular solutions, but also can remove various singularities appearing in the domain integrals. Due to the advantages of RIM that particular solutions are not required and several domain integrals appearing in the same integral equation can be dealt with simultaneously, RIM-based boundary element methods have won a good favour from many BEM researchers [9–12] in recent years. However, although the radial integration boundary element method (RIBEM) is very flexible to deal with the general nonlinear elastic problems [13] and non-homogeneous problems [14–20], there is no report to solve nonlinear heat conduction problems with temperature -dependent conductivity using RIBEM.

In this paper, a new type of boundary-domain integral equation for nonlinear heat conduction problems is developed based on the use of the fundamental solution for linear heat conduction problems for the first time. The resulted domain integrals are transformed to the boundary with the use of RIM by expressing the temperature in the integrand as a series of basis functions. Newton–Raphson iterative method is applied to solve the final system of algebraic equations. Three numerical examples are given to demonstrate the accuracy and efficiency of the present method.

2. Boundary-domain integral equations for heat conduction problems with temperature dependent conductivity

The governing equation for steady state heat conduction problems in isotropic media with temperature dependent thermal conductivity can be expressed as

$$\frac{\partial}{\partial x_i}\left(k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}\right) = 0 \tag{1}$$

where x_i is the i-th component of the spatial coordinates at point $\mathbf{x}$, $T(\mathbf{x})$ the temperature, $k(T(\mathbf{x}))$ the temperature dependent thermal conductivity at point $\mathbf{x}$. The repeated subscript i represents the summation through its range which is 2 for 2D and 3 for 3D problems.

To derive the boundary integral equation, a weight function $G(\mathbf{x},\mathbf{y})$ is introduced to Eq. (1) and the following domain integral can be written:

$$\int_\Omega G(\mathbf{x},\mathbf{y})\frac{\partial}{\partial x_i}\left(k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}\right)d\Omega(\mathbf{x}) = 0 \tag{2}$$

where Ω denotes the domain of the problem of interest.

Using Gauss's divergence theorem, the domain integral can be manipulated as

$$\begin{aligned}\int_\Omega G\frac{\partial}{\partial x_i}\left(k(T)\frac{\partial T}{\partial x_i}\right)d\Omega &= \int_\Gamma Gk(T)\frac{\partial T}{\partial x_i}n_i d\Gamma - \int_\Gamma k(T)T\frac{\partial G}{\partial x_i}n_i d\Gamma \\ &\quad + \int_\Omega T\frac{\partial G}{\partial x_i}\frac{\partial k(T)}{\partial x_i}d\Omega + \int_\Omega k(T)T \\ &\quad \times \frac{\partial}{\partial x_i}\left(\frac{\partial G}{\partial x_i}\right)d\Omega \\ &= 0\end{aligned} \tag{3}$$

where Γ is the boundary of the domain Ω and n_i is the i-th component of outward normal vector $\mathbf{n}$ to the boundary Γ.

If the weight function $G(\mathbf{x},\mathbf{y})$ is chosen as the Green's function which satisfies the following equation:

$$\int_\Omega k(T(\mathbf{x}))T(\mathbf{x})\frac{\partial}{\partial x_i}\left(\frac{\partial G(\mathbf{x},\mathbf{y})}{\partial x_i}\right)d\Omega(\mathbf{x}) = -k(T(\mathbf{y}))T(\mathbf{y}) \tag{4}$$

then by substituting this relation into Eq. (3) and the result into Eq. (2), it follows that

$$\begin{aligned}k(T(\mathbf{y}))T(\mathbf{y}) &= -\int_\Gamma G(\mathbf{x},\mathbf{y})q(\mathbf{x})d\Gamma(\mathbf{x}) - \int_\Gamma k(T(\mathbf{x}))T(\mathbf{x}) \\ &\quad \times \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial \mathbf{n}}d\Gamma(\mathbf{x}) + \int_\Omega \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial x_i} \\ &\quad \times \frac{\partial k(T(\mathbf{x}))}{\partial x_i}T(\mathbf{x})d\Omega(\mathbf{x})\end{aligned} \tag{5}$$

where $q(\mathbf{x})$ is the heat flux:

$$q(\mathbf{x}) = -k(T(\mathbf{x}))\frac{\partial T(\mathbf{x})}{\partial x_i}n_i(\mathbf{x}) \tag{6}$$

The Green's function $G(\mathbf{x},\mathbf{y})$ satisfying Eq. (4) is [7]

$$G(\mathbf{x},\mathbf{y}) = \begin{cases}\frac{1}{2\pi}\ln\left(\frac{1}{r}\right) & \text{for 2D problems} \\ \frac{1}{4\pi r} & \text{for 3D problems}\end{cases} \tag{7}$$

where r is the distance between the source point $\mathbf{y}$ and the field point $\mathbf{x}$.

Eq. (5) is the boundary integral equation for the steady state heat conduction problems with temperature dependent thermal conductivity. In contrast to the conventional linear BEM formulations [1], Eq. (5) includes a domain integral. To evaluate the domain integral, the computational region needs to be discretized into internal cells, which makes BEM lose its distinct advantage of only boundary discretization. To get rid of the cell discretization, the domain integral in Eq. (5) will be transformed into equivalent boundary integral by using RIM in the following section.

3. Transformation of domain integral to the boundary by RIM

In this section, the radial integration method is used to transform the domain integral appearing in Eq. (5) into boundary integral.

In terms of RIM [7,8], a domain integral with the integrand $f(\mathbf{x},\mathbf{y})$ can be transformed into an equivalent boundary integral as follows:

$$\int_\Omega f(\mathbf{x},\mathbf{y})d\Omega(\mathbf{x}) = \int_\Gamma \frac{1}{r^\alpha(\mathbf{z},\mathbf{y})}\frac{\partial r}{\partial n}F(\mathbf{z},\mathbf{y})d\Gamma(\mathbf{z}) \tag{8}$$

where $r(\mathbf{z},\mathbf{y})$ denotes the distance between the source point $\mathbf{y}$ and the boundary point $\mathbf{z}$, and $F(\mathbf{z},\mathbf{y})$ is determined by the following redial integral:

$$F(\mathbf{z},\mathbf{y}) = \int_0^{r(\mathbf{z},\mathbf{y})} f(\mathbf{x},\mathbf{y})r^\alpha dr \tag{9}$$

Fig. 1 shows the relationship among the source point $\mathbf{y}$, field point $\mathbf{x}$, boundary point $\mathbf{z}$ and the distance r.

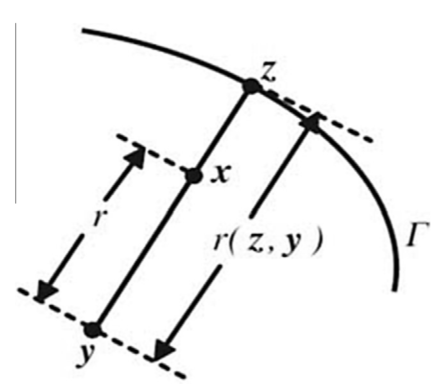

Fig. 1. Relationship among points **x**, **y**, **z** and distances.