International Journal of Heat and Mass Transfer 75 (2014) 610–623

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Mixed convection flow of a nanofluid over a stretching surface with uniform free stream in the presence of both nanoparticles and gyrotactic microorganisms

H. Xu [a,*], I. Pop [b]

[a] State Key Lab of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
[b] Department of Mathematics, Babeş-Bolyai University, R-400084 Cluj-Napoca, Romania

ARTICLE INFO

Article history:
Received 30 September 2013
Received in revised form 19 March 2014
Accepted 31 March 2014
Available online 6 May 2014

Keywords:
Mixed convection
Stretching sheet
Uniform free stream
Nanofluid
Gyrotactic microorganisms
Analytical solution

ABSTRACT

In this paper, a detailed analysis is given for the mixed convection flow of a nanofluid over a stretching surface with uniform free stream in the presence of both nanoparticles and gyrotactic microorganisms. By means of a novel nanofluid model proposed by Kuznetsov and Nield (2013) [24], we reduce the original governing equations embodying the conservation of mass, momentum, thermal energy, nanoparticle volume fraction and conservation equation for microorganisms to a set of five ordinary differential equations with coupled linear boundary conditions. The recursively analytical solutions with high precision are then obtained by the improved homotopy analysis method (HAM). Besides, the influences of important physical parameters such as the Brownian motion parameter N_b, the thermophoresis parameter N_t, the bioconvection Rayleigh number R_b, the bioconvection Péclet number P_e on the local wall friction, the local Nusselt number, the local wall mass flux, the local wall motile microorganisms flux, as well as the profiles of velocity, temperature, nanoparticles concentration and density of motile microorganisms are examined and discussed in detail.

1. Introduction

Flow and heat transfer in the boundary layer over a continuous solid surface is frequently encountered in many industrial and engineering applications such as materials manufactured by extrusion processes and heat-treated materials traveling between a feed roll and a wind-up roll or on a conveyor belt possess the characteristics of a moving continuous surface. Since Sakiadis [1] initiated the study of boundary layer flow over a flat surface with a constant speed, many researchers successively investigated various kinds of the boundary layer flow due to a continuously moving or stretching surface. Among those work, Erickson et al. [2] extended Sakiadis' problem to the case that either suction or injection is allowed through the moving wall and considered its effects on the flow and heat transfer in the boundary layer. Different from Sakiadis's case [1], Crane [3] made an analysis for a boundary layer flow due to a flat stretching surface whose velocity is proportional to the distance from the leading edge of the slit. Danberg and Fansler [4] investigated a boundary layer flow over a continuous moving wall with the wall velocity being a constant and the outer edge velocity being linear to the distance from the slit. Gupta and Gupta [5] analyzed a heat fluid flow past an isothermal stretching surface in the presence of wall blowing or suction. Chen and Char [6] presented an analysis for heat transfer behaviors of a boundary layer flow over a linearly stretching, porous surface with the prescribed wall temperature or heat flux. Vajravelu and Hadjinicolaou [7] considered a laminar flow and heat transfer of an electrically conducting fluid near a flat sheet stretched linearly with a velocity proportional to the distance along the wall in the presence of a uniform free stream of constant velocity and temperature. Wang [8] made an extension of Crane's problem [3] to the case that the flat sheet is stretched linearly in two lateral directions in an otherwise ambient fluid and obtained exact similarity solutions for this particular fluid flow. Takhar et al. [9] further extended Wang's problem [8] by considering the unsteady effect on the three-dimensional boundary layer flow in the presence of a magnetic field. Very recently, Khan and Pop [10] carried out an analysis of a nanofluid flow and heat transfer in the boundary layer region of a linearly stretched flat surface and presented a numerical solution for the velocity and temperature field.

Recently, some researchers began to pay their attention to the problems of hydrodynamic and thermal boundary layers subjected to the slip wall boundary conditions. Those slip boundary

* Corresponding author. Tel.: +86 21 34204407; fax: +86 21 34207991.
E-mail addresses: hangxu@sjtu.edu.cn (H. Xu), popm.ioan@yahoo.co.uk (I. Pop).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2014.03.086
0017-9310/

conditions are often used for nanofluid flows near a solid wall and are found to be very appropriate for describing the relationship between the base fluid velocity and the slip velocity due to the nanoparticles. Several models have been proposed for prediction the behavior of nanofluids, such as the homogenous model [11,12], the dispersion model [13], the Buongiorno model [14]. And some critical reviews [15–19] have been carried out towards the understandings of the transport phenomena of various kinds of nanofluids. Among those models, the Buongiorno model [14] attracted more and more attention since it provided a very reasonable description for the mechanism of the nanoparticle/base-fluid relative velocity. In the absence of turbulent effects, Buongiorno [14] concluded that the Brownian diffusion and the thermophoresis are the most important factors to alter the behaviors of the nanofluid flows. Kuznetsov and Nield [20] examined the influence of nanoparticles on a free convection flow over a vertical flat surface with the Buongiorno model in which Brownian motion and thermophoresis are accounted for. Nield and Kuznetsov [21] made an extension to the well-known Cheng–Minkowycz [22] problem by considering the natural convection nanofluid flow past a vertical plate in a saturated porous medium. Xu et al. [23] studied a fully developed mixed convection flow in a vertical channel filled by a nanofluid in the presence of buoyancy force due to the linearly heated walls. Very recently, Kuznetsov and Nield [24] further improved on the Buongiorno model by assuming that the nanofluid particle fraction on the boundary is passively rather than actively controlled. This makes the model physically more realistic than the previous one used by some authors [20,21,10,23]. Nanofluids have great potential in applications of microfluidic devices since they are very helpful for mass transport enhancement, induce mixing, especially in microvolumes, as well as improve on the stability of nanofluids. A novel idea for design of next generation of microfluidic devices is to combine a nanofluid with bioconvection. Thus studies of bioconvection in nanofluids must be investigated fundamentally. Kuznetsov and Avramenko [25] was the first to examine the effect of solid particles on the stability of the bioconvection with a suspension of motile gyrotactic microorganisms in a horizontal fluid layer of finite depth. Kuznetsov and Geng [26] further studied the influence of settling of small solid particles in a dilute suspension that contains both gyrotactic micro-organisms and solid particles on the bioconvection in a chamber of finite depth. Kuznetsov [27] analyzed the onset of bioconvection in a horizontal layer filled with both the naoparticles and gyrotactic microorganisms. Kuznetsov [28] presented an analysis on non-oscillatory and oscillatory bioconvection of a nanofluid in a horizontal layer of finite depth in the present of both nanoparticles and gyrotactic microorganisms. Kuznetsov and Bubnovich [29] investigated the simultaneous effects of gyrotactic and oxytactic microorganisms on nanofluid biothermal convection in a porous medium.

This paper intends to incorporate the newly developed nanofluid model proposed by Kuznetsov and Nield [24] to the bioconvection in a suspension containing both nanoparticles and gyrotactic microorganism. The major problems of the current model with regarding to the damage due to the Joule heating generated by active mixes could be well resolved by employing such passively controlled model. Thus, the physical mechanism of nanofluids bioconvection involving into the slip velocity between the nanoparticles and the base fluid is expected to be physically more realistic than the previous ones. The classic problem of mixed convection flow over a vertical flat sheet in the presence of a uniform free stream will be extended to the case that both the nanoparticles and gyrotactic microorganisms are included into the base fluid. With the boundary layer approximations, the conservation equations for momentum, energy, nanoparticles and microorganisms are reduced to a set of five fully coupled differential equations with linear boundary conditions. The complex nonlinear system will then be solved analytically by the homotopy analysis method with a newly developed technique for complicated boundary conditions. As far as we know, this problem has not been studied before so that the results are novel and original.

2. Flow analysis

In natural circumstance, the microorganisms can only be survival in water. Hence the base fluid to be considered has to be water. It is assumed that the suspended nanoparticles can be kept stably and do not agglomerate in the fluid. It is also assumed that the nanoparticles has no influence on the swimming direction and velocity of the microorganisms. To avoid bioconvection instability due to the increase of the suspension's viscosity, the suspension of nanoparticles in the fluid has to be dilute. With those assumptions, the following five field equations which embody the conservation of total mass, momentum, thermal energy, nanoparticle volume fraction and microorganisms can be obtained, using the nanofluid model proposed by Kuznetsov and Nield [24], in the following forms:

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho_f(\mathbf{v}\cdot\nabla)\cdot\mathbf{v} = -\nabla p + \mu\nabla^2\mathbf{v} + \Big\{C\rho_p + (1-C)\Big[\rho_f(1-\beta(T-T_\infty))\Big] + \gamma\Delta\rho N\Big\}\mathbf{g}, \tag{2}$$

$$\mathbf{v}\cdot\nabla T = \alpha\nabla^2 T + \tau[D_B\nabla T\cdot\nabla C + (D_T/T_\infty)\nabla T\cdot\nabla T], \tag{3}$$

$$(\mathbf{v}\cdot\nabla)C = D_B\nabla^2 C + (D_T/T_\infty)\nabla^2 T, \tag{4}$$

$$\nabla\cdot\mathbf{j} = 0, \tag{5}$$

where $\mathbf{v} = (u, v, w)$ is the velocity of nanofluid flow, p is the pressure, T is the temperature, C is the nanoparticle concentration, N is the concentration of microorganisms, $\mathbf{j}$ is the flux of microorganisms due to fluid convection, self-propelled swimming, and diffusion, ρ_f is the density of nanofluid, ρ_p is the density of the nanoparticles, T_∞ is the reference temperature, μ is the viscosity of the suspension of nanofluid and microorganisms, β is the volumetric coefficient of thermal expansion of the base fluid, $\mathbf{g}$ is the gravity vector, γ is the average volume of a microorganism, $\Delta\rho = \rho_{\text{cell}} - \rho_{bf}$ is the density difference between a cell and a base fluid, α is the thermal diffusivity of the nanofluid, $\tau = (\rho c)_p/(\rho c)_f$ with $(\rho c)_p$ being heat capacity of the nanoparticle and $(\rho c)_f$ being the heat capacity of fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient. It should be noted here that the Oberbeck–Boussinesq approximation has been applied for linearization of the buoyancy force in Eq. (2) with the term $\rho_{f\infty}\beta(T-T_\infty)C$ being neglected. This assumption is available for a dilute suspension nanoparticles so that the theory for gyrotactic microorganisms in Refs. [30,31] can be still used here. Furthermore, the requirement of the small temperature gradient across the layer can ensure that the microorganisms can be alive in the fluid with suspension of nanoparticles.

In Eq. (5), the flux of microorganisms $\mathbf{j}$ is defined as

$$\mathbf{j} = N\mathbf{v} + N\hat{\mathbf{v}} - D_n\nabla N, \tag{6}$$

where

$$\hat{\mathbf{v}} = (bW_c/\Delta C)\nabla C, \tag{7}$$

in which, D_n is the diffusivity of microorganisms, b is the chemotaxis constant $[m]$ and W_c is the maximum cell swimming speed $[m/s]$ (the product bW_c is assumed to be constant).

Invoking the boundary layer approximations, the governing equations describing the two-dimensional flow and heat transfer