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Phase-field simulation of Rayleigh instability on a fibre

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ABSTRACT

In this paper, we present a phase-field method for Rayleigh instability on a fibre. Unlike a liquid column, the evolutionary dynamics of a liquid layer on a fibre depends on the boundary condition at the solid-liquid interface. We use a Navier–Stokes–Cahn–Hilliard system to model axisymmetric immiscible and incompressible two-phase flow with surface tension on a fibre. We solve the Navier–Stokes equation using a projection method and the Cahn–Hilliard equation using a nonlinearly stable splitting method. We present computational experiments with various thicknesses of liquid thread and fibre. The numerical results indicate that the size of the satellite droplet decreases as the thicknesses of the thread and fibre increase.

1. Introduction

The coating problem of a cylindrical fibre with a liquid film has been intensively studied because of its relation with technological and industrial processes, i.e., the coating of conducting cables with isolating films (González et al., 2010). However, compared to a lot of literature on the simulations of the break-up of a liquid thread under the Rayleigh instability (RI) (Chakrabarti et al., 2017; Gopan and Sarith, 2014; Joshi et al., 2016; Vega et al., 2010; Yan et al., 2015) and references therein, there are only few numerical works on the RI on a fibre (González et al., 2010; Haefner et al., 2015; Mead-Hunter et al., 2012). Fig. 1 shows optical micrographs illustrating the temporal evolutions of the Plateau–Rayleigh instability for a liquid polystyrene film on a glass fibre (Haefner et al., 2015).

Using a lubrication approximation, the authors in Haefner et al. (2015) obtained a governing equation for the one-dimensional axisymmetric surface profile over time and compared with various experiments. In particular, they reported on the RI dynamics with two different boundary conditions on the liquid and fibre. The breakup of a liquid film coating a fiber into an array of droplets was simulated using a three-dimensional volume-of-fluid method by the authors in Mead-Hunter et al. (2012). They also compared the numerical results with experimental observations and existing theory. The instability of a liquid film coating a thin cylindrical fibre was investigated numerically and experimentally by González et al. (2010). They reported experimental results such as growth rates and dominant wavelengths of the interface. They also presented direct numerical simulations and compared with the experimental data.

In this work, we will use a phase-field method for the Rayleigh instability on a fibre and investigate the effects of the thickness of the liquid film and the fibre on the evolution dynamics. The phase-field method is popular in modeling two-phase fluid flows. For example, Bai et al. (2017) used a 3D phase-field model to simulate the droplet formation process in a flow-focusing device. There are other numerical methods for multiphase fluid flows such as the level-set method (Rodríguez, 2017) and the volume-of-fluid method (Müller et al., 2016).

The outline of the paper is as follows. The phase-field model in cylindrical coordinates is presented in Section 2. The numerical solution is given in Section 3. The proposed numerical schemes are tested in Section 4. Finally, conclusions are derived in Section 5.

2. Axisymmetric Navier–Stokes–Cahn–Hilliard system

We consider the two-phase fluid consisting of two components, fluid 1 and fluid 2, on a solid fibre. We denote by ϕ the composition difference of the mixture of two fluids. The phase-field ϕ is a normalized concentration and its value is equal to $+1$ and -1 when the two phases are at mutual equilibrium. In this study, we focus on density matched case. The axisymmetric Navier–Stokes–Cahn–Hilliard system (Kim, 2005b) is

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\rho(\mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \nabla \cdot [\eta(\phi)(\nabla \mathbf{u} + \nabla \mathbf{u}^T)] + \mathbf{SF}(\phi), \tag{2}$$

$$\phi_t + \nabla \cdot (\phi \mathbf{u}) = M \Delta \mu, \tag{3}$$

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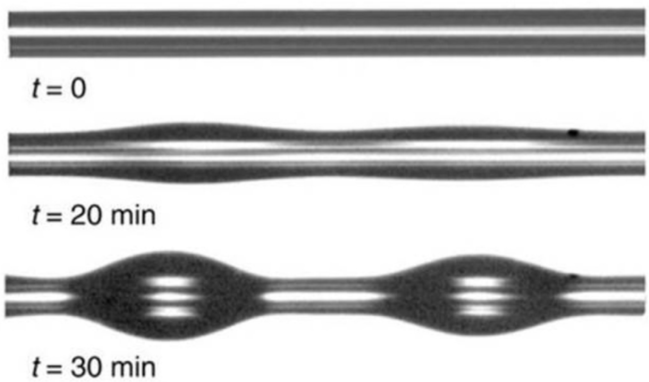

Fig. 1. Plateau–Rayleigh instability for a liquid polystyrene film on a glass fibre. Adapted from Haefner et al. (2015) with permission from Nature Publishing Group.

$$\mu = \alpha\phi^3 - \beta\phi - \kappa\Delta\phi, \tag{4}$$

where $\mathbf{u}$ the velocity, ρ is the density, p the pressure, and $\eta(\phi) = \eta_1(1+\phi)/2 + \eta_2(1-\phi)/2$ is the variable viscosity, where η_1 and η_2 are viscosity coefficients of fluid 1 and 2, respectively. The surface tension force (Kim, 2005a) is

$$\mathbf{SF}(\phi) = -\frac{3\sqrt{2}\sigma\epsilon}{4}\nabla\cdot\left(\frac{\nabla\phi}{|\nabla\phi|}\right)|\nabla\phi|\nabla\phi, \tag{5}$$

where σ is the interfacial tension coefficient and ϵ is the small positive parameter related to interfacial transition thickness. Other thermodynamically consistent surface tension force, i.e., the Korteweg force can be found in Kim (2005a) and Lamorgese et al. (2017), M is the positive mobility, and α, β, κ are constant. μ is the generalized (i.e., including its non-local part) chemical potential difference near the critical point. Eqs. (1)–(5) couple each other through concentration-dependent viscosity and surface tension force, and the advection for the phase-field ϕ. If we nondimensionalize the governing Eqs. (1)–(4), then we have

$$\nabla\cdot\mathbf{u} = 0, \tag{6}$$

$$\mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \frac{1}{Re}\nabla\cdot[\eta(\phi)(\nabla\mathbf{u} + \nabla\mathbf{u}^T)] + \frac{1}{We}\mathbf{SF}(\phi), \tag{7}$$

$$\phi_t + \nabla\cdot(\phi\mathbf{u}) = \frac{1}{Pe}\Delta\mu, \tag{8}$$

$$\mu = \phi^3 - \phi - \epsilon^2\Delta\phi, \tag{9}$$

where we set $\alpha, \beta = 1$, $\kappa = \epsilon^2$ and the Reynolds, Weber, and Peclet numbers are given by $Re = \rho_c U_c L_c/\eta_c$, $We = \rho_c L_c U_c^2/\sigma_c$, and $Pe = U_c L_c/(M_c\mu_c)$ using characteristic values, respectively. More details about the nondimensionalization can be found in Kim (2005b) and Lee et al. (2011). Because we are interested in axisymmetric Navier–Stokes–Cahn–Hilliard system for the cylindrical viscous liquid thread on a solid fibre, we rewrite Eqs. (6)–(9) in axisymmetric form:

$$\frac{1}{r}(ru)_r + w_z = 0, \tag{10}$$

$$\begin{aligned} u_t + uu_r + wu_z = &-p_r + \frac{SF^r}{We} \\ &+ \frac{1}{Re}\left(\frac{1}{r}(r(2\eta u_r))_r + (\eta(w_r + u_z))_z - \frac{2\eta u}{r^2}\right), \end{aligned} \tag{11}$$

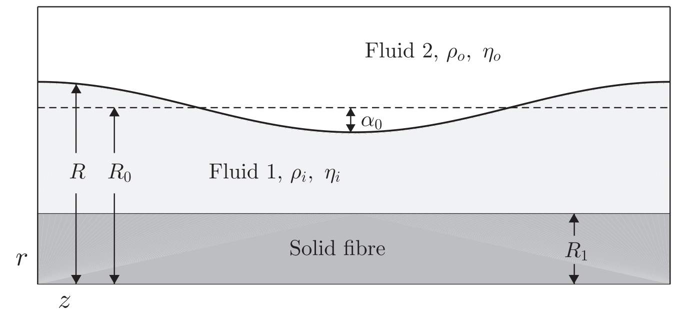

Fig. 2. Schematic of a perturbed cylindrical thread of viscous fluid 1 coating a fibre and embedded in another viscous fluid 2.

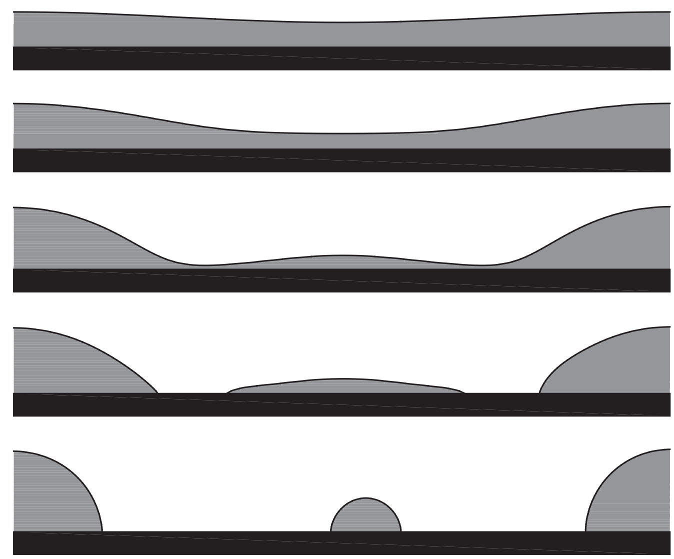

Fig. 3. Temporal evolution of a thread coating a fibre.

$$\begin{aligned} w_t + uw_r + ww_z = &-p_z + \frac{SF^z}{We} \\ &+ \frac{1}{Re}\left(\frac{1}{r}(r\eta(w_r + u_z))_r + (2\eta w_z)_z\right), \end{aligned} \tag{12}$$

$$\phi_t + \frac{1}{r}(r\phi u)_r + (\phi w)_z = \frac{1}{Pe}\left(\frac{1}{r}(r\mu_r)_r + \mu_{zz}\right), \tag{13}$$

$$\mu = \phi^3 - \phi - \epsilon^2\left(\frac{1}{r}(r\phi_r)_r + \phi_{zz}\right), \tag{14}$$

where $u = u(r, z)$ and $w = w(r, z)$ are the radial and the axial velocities, respectively. The subscript index is the differentiation with respect to that index.

3. Numerical solution

Let us consider a two-dimensional axisymmetric computational domain $\Omega = \{(r, z) : R_1 < r < R_2,\ 0 < z < H\}$. We discretize the domain with a uniform mesh spacing h. The center of each cell is positioned at $(r_i, z_k) = (R_1 + (i - 0.5)h, (k - 0.5)h)$ for $i = 1, \cdots, N_r$ and $k = 1, \cdots, N_z$, where N_r and N_z are the numbers of cells in r and z-directions, respectively. The cell vertices are located at $(r_{i+\frac{1}{2}}, z_{k+\frac{1}{2}}) = (R_1 + ih, kh)$. Given $\mathbf{u}^n = (u^n, w^n)$ and ϕ^n, we want to find $\mathbf{u}^{n+1} = (u^{n+1}, w^{n+1})$ and p^{n+1} which solve the following discrete Eqs. (10)–(12):

$$\frac{1}{r}(ru^{n+1})_r + w_z^{n+1} = 0, \tag{15}$$