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An analytical film drainage model and breakup criterion for Taylor bubbles in slug flow in inclined round pipes

E. Lizarraga-Garcia[a,*], J. Buongiorno[b], M. Bucci[b]

[a] Department of Mechanical Engineering, MIT, Cambridge, MA 02139, USA
[b] Department of Nuclear Science and Engineering, MIT, Cambridge, MA 02139, USA

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ABSTRACT

The velocity of Taylor bubbles in inclined pipes is reduced if a lubricating liquid film between the bubble and the pipe wall is not present. An analytical model predicting the gravity-driven drainage of the lubricating film is presented in this article. The model is then used to establish a criterion for film breakup: if $\bar{t}_{bubble} = t_{bubble}/\tau < 0.01$ the thin film would not break up, where t_{bubble} is the bubble's passage time, and τ is the characteristic film drainage time based on the fluid properties, pipe geometry, and critical film thickness. The model is validated experimentally with Taylor bubbles in inclined pipes (5° to 90°, the latter being vertical) with stagnant liquids (ethanol, methanol, and mixtures of deionized water and methanol).

1. Introduction

Flow of Taylor bubbles, also known as slug flow, is a common occurrence in wells, riser pipes and pipelines of crude oil and natural gas developments, as well as boiling-water nuclear reactors. Current predictive methods for this flow pattern rely on the so-called mechanistic two-fluid model, where the flow is represented as a series of liquid slugs and Taylor bubbles (Taitel and Dukler, 1976; Orell and Rembrand, 1986; Ansari et al., 1994; Petalas and Aziz, 2000). For the case of vertical pipes, an axisymmetric lubricating film with a constant thickness surrounds the Taylor bubble. For stagnant liquid, the range of the non–dimensional film thickness, $\bar{h} = h/R$, where h is the film thickness and R is the pipe radius, is approximately $\bar{h} \in [0.08, 0.33]$ (Llewellin et al., 2012). As the pipe inclination increases, the Taylor bubble approaches the pipe wall and the lubricating film becomes significantly thinner and non-axisymmetric; moreover, the thickness of the film decreases along the Taylor bubble due to azimuthal gravity-driven drainage (see Fig. 1). If the film breaks up, the surface tension force at the triple contact line reduces the velocity of the bubble significantly (Behafarid et al., 2015).

The existence of this lubricating film and its breakup have received some attention in the literature. Maneri and Zuber (1974) and Hien and Fabre (2004a) studied the velocity of plane bubbles in two-dimensional ducts experimentally and numerically, respectively, using deionized (DI) water and methanol. They observed three different bubble shape regimes depending on the duct inclination: (i) the bubble touching the upper wall for $\theta \leq 60°$, (ii) a stable lubricating film where the bubble does not touch the duct for $\theta \geq 80°$, and (iii) an unstable transition region in between. Al-Safran et al. (2013) observed a stable thin film at the top of the horizontal pipe in their slug flow experiments with high-viscosity fluids. However, these results are valid for the limited set of fluid properties and flow conditions explored in those studies. The drainage of a vertical film due to gravity was analyzed by Mysels et al. (1959); here we extend the analysis to the situation where the component of gravity in the direction of the flow varies continuously, and surface tension and intermolecular forces may affect the dynamics (Oron et al., 1997).

In this article, a drainage model and breakup criterion for the lubricating film of Taylor bubbles in slug flow in inclined round pipes is presented. Such criterion can be used to determine under which conditions the lubricating film is present, which is a key input for both numerical simulations (Hien and Fabre, 2004b; Taha and Cui, 2006; Ben-Mansour et al., 2010; Lizarraga-Garcia et al., 2015b) and mechanistic modeling of slug flow in order to determine correctly the Taylor bubble velocity and pressure drop. Also, it can be applied in flow assurance studies of high-viscosity oil slug flows, a critical aspect in oil and gas systems: corrosion of the pipe material causes its blockage, and antioxidants are added to the liquid to avoid it. The prediction of a liquid film above the Taylor bubble so that antioxidants touch the entire pipe is thus key to guarantee their safety.

* Corresponding author.
E-mail address: elizaga@mit.edu (E. Lizarraga-Garcia).

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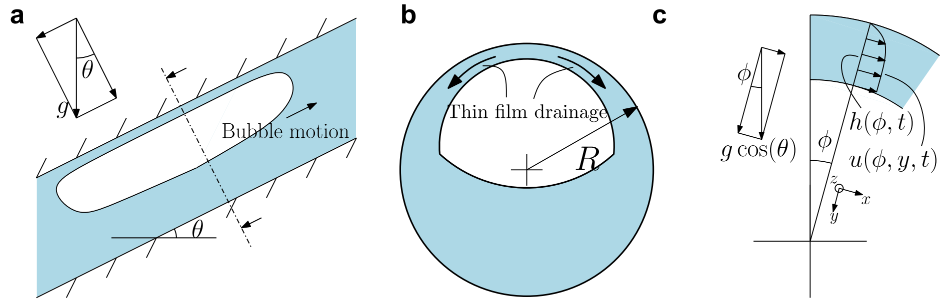

Fig. 1. Taylor bubble and lubricating liquid film inside a round pipe with a θ inclination angle with respect to the horizontal (a), cross-sectional view (b), and coordinates used for the film drainage analysis (c). Not drawn to scale.

2. Development of the thin film drainage and breakup model

2.1. Film drainage

Fig. 1 shows the geometry and frame of reference chosen for the analysis of the lubricating liquid film drainage. Let u, v, and w denote the liquid film velocity in the azimuthal, radial, and longitudinal direction, respectively. Use of Cartesian coordinates is justified since $h/R \ll 1$. Thus, the Navier–Stokes equation in the x direction is

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\rho w\frac{\partial u}{\partial z}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+F_x(\phi), \tag{1}$$

where μ is the liquid viscosity, ρ is the liquid density, p is the pressure, $F_x(\phi)=\rho g\cos(\theta)\sin(\phi)$ where g is the gravity acceleration, and ϕ is the azimuthal angle with respect to the vertical. Eq. (1) can be simplified using the lubrication approximation, by virtue of which various terms can be neglected:

$$\frac{\partial^2 u}{\partial x^2}\Big/\frac{\partial^2 u}{\partial y^2}\ll 1, \tag{2a}$$

$$\frac{\partial^2 u}{\partial z^2}\Big/\frac{\partial^2 u}{\partial y^2}\ll 1, \tag{2b}$$

$$\rho u\frac{\partial u}{\partial x}\Big/\mu\frac{\partial^2 u}{\partial y^2}\ll 1, \tag{2c}$$

$$\rho\frac{\partial u}{\partial t}\Big/\mu\frac{\partial^2 u}{\partial y^2}\ll 1. \tag{2d}$$

Note that Eq. (2c) applies equally to the other two inertia terms of the equation after the continuity equation. Also, the pressure term can be neglected,

$$\frac{\partial p}{\partial x}\approx 0, \tag{3}$$

considering that the pressure differences inside the film due to gravity and surface tension are negligible, and the pressure inside the bubble is constant. Furthermore, the intermolecular forces are not included in Eq. (1). The validity of these approximations is verified in Appendix A. Thus, the previous Navier–Stokes Eq. (1) is simplified and, after imposing the no-slip at the wall and shear-stress-free at the film surface boundary conditions, the azimuthal film velocity profile is found:

$$u(\phi,y)=\frac{F_x(\phi)}{\mu}\left(hy-\frac{y^2}{2}\right), \tag{4}$$

a parabolic profile whose approximate shape is depicted in Fig. 1c. Similarly, the Navier–Stokes equation in the z direction is

$$\rho\frac{\partial w}{\partial t}+\rho u\frac{\partial w}{\partial x}+\rho v\frac{\partial w}{\partial y}+\rho w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+F_z, \tag{5}$$

where $F_z=\rho g\sin(\theta)$. Note that v is much smaller than the other two velocity terms in this lubrication approximation. Following an analogous procedure as in the x direction, Eq. (5) is simplified and we obtain the longitudinal film velocity profile,

$$w(\phi,y)=\frac{F_z}{\mu}\left(hy-\frac{y^2}{2}\right). \tag{6}$$

In order to obtain the governing PDE for the film drainage, the continuity equation is used:

$$\frac{\partial h}{\partial t}+\frac{\partial Q'_x}{\partial x}+\frac{\partial Q'_z}{\partial z}=\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}\int_0^h u\,dy+\frac{\partial}{\partial z}\int_0^h w\,dy=0, \tag{7}$$

where Q'_x and Q'_z are the volumetric flow per unit length in the x and z direction, respectively. Using Eqs. (4) and (6), and recognizing that $x=\phi\cdot R$, the second and third terms of the LHS of the previous equation can be developed:

$$\frac{\partial Q'_x}{\partial x}=\frac{\rho g\cos(\theta)}{\mu}h^2\sin\left(\frac{x}{R}\right)\frac{\partial h}{\partial x}+\frac{\rho g\cos(\theta)}{3\mu R}h^3\cos\left(\frac{x}{R}\right), \tag{8a}$$

$$\frac{\partial Q'_z}{\partial z}=\frac{\rho g\sin(\theta)}{\mu}h^2\frac{\partial h}{\partial z}. \tag{8b}$$

Noting that the three RHS terms of the previous equations are positive, and $(\partial h/\partial z)/(\partial h/\partial x)\ll 1$ by scaling analysis it can be concluded that

$$\frac{\partial Q'_z}{\partial z}\ll\frac{\partial Q'_x}{\partial x}, \tag{9}$$

and therefore the film drainage PDE becomes

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial\phi}\left(\frac{\rho g\cos(\theta)h^3}{3\mu R}\sin(\phi)\right)=0. \tag{10}$$

The initial and boundary conditions are

$$h(\phi,0)=h_i(\phi), \tag{11a}$$

$$\frac{\partial h(0,t)}{\partial\phi}=0, \tag{11b}$$

respectively, where Eq. (11b) comes from the solution's symmetry at $\phi=0$. An analytical solution for Eq. (10) can be obtained at $\phi=0$ using the method of characteristics through the Lagrange–Charpit equations (Delgado, 1997). After some simple algebra, the thin film drainage at $\phi=0$ is

$$h(\phi=0,t)=\left(\frac{1}{h_0^2}+\frac{2\rho g\cos(\theta)}{3\mu R}t\right)^{-1/2}, \tag{12}$$