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# Stability of multiple solutions in inclined gas/shear-thinning fluid stratified pipe flow

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ABSTRACT

This work provides an investigation on multiple solutions in gas/shear-thinning fluid inclined stratified pipe flows. Multiple solution operative conditions are studied investigating the effect of the interfacial shear stress modeling and the rheology of the shear-thinning fluid. The modeling of the interfacial shear stress in counter-current has a strong influence of multiple solutions regions. The stability of multiple hold-up solutions is studied considering the structural stability, the interfacial stability, and the minimization of the dissipation approaches. The results of the three different approaches are commented both for concurrent and counter-current flows, giving the same conclusions only for upward inclined flows.



## 1. Introduction

Gas/liquid stratified flow is the basic horizontal pipe flows in chemical and petroleum industry.

Multiple hold-up solutions in inclined pipe flows have been widely studied by Landman (1991), Barnea and Taitel (1992), and Brauner (2002) for Newtonian/Newtonian fluid systems, while the presence of multiple solutions for gas/shear-thinning fluid stratified pipe flows has not been investigated yet.

Despite the interest in non-Newtonian conditions only few works have been developed. The two-fluid model by Taitel and Dukler (1976) was first extended to gas/power-law fluid pipe flow by Heywood and Charles (1979) to horizontal flows and, only more recently, to inclined pipes by Xu et al. (2007), where the existence of multiple solutions for inclined flows was only mentioned. In addition, Picchi et al. (2014) proposed a pre-integrated model and carried out an interfacial linear stability and well-posedness analyses on the governing equations of gas/shear-thinning fluid stratified flow to determine the transition boundaries from the stratified flow regime.

Experiments on gas/shear thinning stratified flows were carried out by Bishop and Deshpande (1986), Xu et al. (2007), Picchi et al. (2015), where hold-up, pressure drop measurements, and flow pattern maps were presented, but multiple solutions are not observed for the investigated conditions. Jia et al. (2011) presented numerical simulations, where the presence of multiple hold-up solutions is only mentioned.

When multiple hold-up solutions exist, the interest focuses on which of them is a stable configuration for the system and, therefore, can be considered feasible. Concerning Newtonian/Newtonian fluid stratified flows, different approaches are present in the literature. Barnea and Taitel (1992, 1994b, 1994a) carried a structural stability analysis on upward inclined pipe flows. Ullmann et al. (2003b, 2003c), Kushnir et al. (2014), and Thibault et al. (2015) discussed the presence of multiple solutions in concurrent and counter-current laminar–laminar stratified flow in a two-plate geometry, proposing other three approaches: the long-wave interfacial stability (Kushnir et al., 2014), the catastrophe theory (Thibault et al., 2015), and the minimization of the dissipation rate (Poesio and Beretta, 2008). Goldstein et al. (2015) studied multiple solutions for laminar-laminar stratified and fully eccentric core-annular flows in pipes considering the exact solution for the velocity profiles both in the concurrent and counter-current cases.

In this paper, we discuss multiple hold-up solutions for inclined gas/shear-thinning stratified pipe flow (Section 3.1) investigating the effect of the rheology (Section 3.1.1) and of the interfacial shear stress modelling (Section 3.1.2). Then, we investigate the stability of multiple solutions considering the structural stability (linear and non-linear), the long wave interfacial stability (linear) (Section 3.2), and the minimization of the dissipation approach (Section 3.3) to determine which configuration can be considered feasible for the case of concurrent and counter-current inclined flows.

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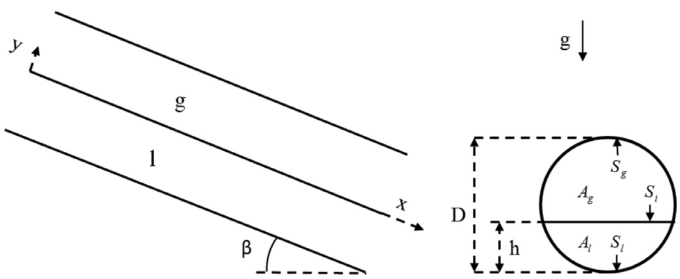


**Fig. 1.** Sketch of the two-phase flow pipe geometry: the pipe (or channel for the pre-integrated model) flow on the left and the pipe flow cross section on the right.

## 2. Theoretical considerations

### 2.1. Two-fluid model for gas/shear-thinning fluid stratified flow

The two-fluid model governing equations for a gas/power-law liquid stratified flow (Fig. 1) in a horizontal or slightly inclined pipe, considering incompressible fluids, velocity shape factors equal to unity ($\gamma_g = \gamma_l = 1$), yield (Picchi et al., 2014),

$$\frac{\partial}{\partial t}A_l + \frac{\partial}{\partial x}(A_l U_l) = 0, \tag{1a}$$

$$\frac{\partial}{\partial t}A_g + \frac{\partial}{\partial x}(A_g\, U_g) = 0, \tag{1b}$$

$$\begin{aligned}\rho_l \frac{\partial U_l}{\partial t} + \rho_l U_l \frac{\partial U_l}{\partial x} - \rho_g \frac{\partial U_g}{\partial t} - \rho_g U_g \frac{\partial U_g}{\partial x} \\ +(\rho_l - \rho_g) g \cos\beta \frac{\partial h}{\partial x} - \sigma \frac{\partial^3 h}{\partial x^3} = F,\end{aligned} \tag{2a}$$

$$F = -\tau_l \frac{S_l}{A_l} + \tau_i S_i \left( \frac{1}{A_g} + \frac{1}{A_l} \right) + \tau_g \frac{S_g}{A_g} + (\rho_l - \rho_g) g \sin\beta, \tag{2b}$$

where $A$, $S$, $\tau$, $\rho$, and $U$ are the flow cross section, the wetted perimeter, the shear stress, and the average velocity of the two fluids, respectively. The subscripts $g$ and $l$ refer to gas and liquid, respectively. $\beta$ is the pipe inclination angle (a positive $\beta$ corresponds to downward inclined flow) and $\sigma$ is the surface tension. Eqs. (1) and (2) are valid both for concurrent and counter-current flows; for counter-current flows $U_l$ assumes negative values.

The shear stresses are evaluated, considering the liquid as a power-law fluid, as

$$\tau_g = f_g \frac{\rho_g U_g |U_g|}{2}, \quad f_g = C_g Re_g^{-m_g}, \quad Re_g = \frac{\rho_g |U_g| D_g}{\mu_g}, \tag{3a}$$

$$\tau_l = f_l \frac{\rho_l U_l |U_l|}{2}, \quad f_l = C_l Re_l^{-m_l}, \quad Re_l = \frac{D_l^n |U_l|^{2-n} \rho_l}{m 8^{n-1} \left(\frac{1+3n}{4n}\right)^n} = \frac{\rho_l |U_l| D}{\mu_{app}}. \tag{3b}$$

where $\mu_{app}$, $m$, and $n$ are the apparent viscosity, the fluid consistency index, and the fluid behavior index for a power-law fluid. The constants are chosen as $C_g = C_l = 16$, $m_g = m_l = 1$ for the laminar flow regime and as $C_g = C_l = 0.046$, $m_g = m_l = 0.2$ for the turbulent flow regime; for the power-law liquid laminar-turbulent transition we follows Chhabra and Richardson (2008).

The equivalent hydraulic diameters are evaluated for concurrent flows as

$$D_g = \frac{4A_g}{S_i + S_g}, \quad D_l = \frac{4A_l}{S_l}, \quad \text{if} \;\; U_g > U_l, \tag{4a}$$

$$D_g = \frac{4A_g}{S_g}, \quad D_l = \frac{4A_l}{S_l + S_i}, \quad \text{if} \;\; U_g < U_l; \tag{4b}$$

for counter-current flows each of the layers is considered dragged by the other one, see Ullmann et al. (2003b), yielding

$$D_g = \frac{4A_g}{S_i + S_g}, \quad D_l = \frac{4A_l}{S_l + S_i}. \tag{5}$$

Specific closures for the interfacial shear stress are not available in the literature for gas/power-law fluid stratified flows, thus $\tau_i$ is calculated as for air/Newtonian fluid stratified flows, yielding

$$\tau_i = f_i \frac{\rho_i (U_g - U_l)|U_g - U_l|}{2}. \tag{6}$$

We consider the interfacial friction factor $f_i$ in concurrent flows as $f_i = f_g$ and $\rho_i = \rho_g$ if $U_g > U_l$, and $f_i = f_l$ and $\rho_i = \rho_l$ if $U_g < U_l$. This approach is used by Heywood and Charles (1979) and Picchi et al. (2014) in the case of $U_g > U_l$; here, we considered also the case when the interfacial shear stress is controlled by the liquid phase ($U_l > U_g$) like in the case of multiple solution conditions in downward inclined flows, where the liquid is the faster phase. To perform a sensitivity analysis of the effect of the different $f_i$ correlations on the multiple solution boundaries, we tested also (see Section 3.1.2) the following modeling:

- $f_i = 0.014$ and $\rho_i = \rho_g$ as suggested by Cohen and Hanratty (1968) and used by Xu et al. (2007), considering only the case where $U_g > U_l$.
- Jia et al. (2011) proposed to evaluate the interfacial friction factor with Andreussi and Persen (1987) empirical correlation

$$f_i/f_g = \begin{cases} 1.0 & if \;\; F_{AP} \le 0.36, \\ 1.0 + 29.7(F_{AP} - 0.36)^{0.67} \left(\dfrac{h}{D}\right)^{0.2} & if \;\; F_{AP} > 0.36, \end{cases} \tag{7}$$

  where the dimensionless number $F_{AP} = U_g \sqrt{\left(\frac{\rho_g}{\rho_l - \rho_g} \frac{dA_l/dh}{A_g} \frac{1}{g\cos\beta}\right)}$; this experimental correlation is valid for $U_g > U_l$.
- Picchi et al. (2014) presented a pre-integrated model for turbulent gas/laminar power-law stratified flow: a correction for the liquid hydraulic diameter is presented considering the flow of an equivalent two-plate stratified flow.

For counter-current flows, Ullmann et al. (2003b) discussed the choice of $f_i$ for the two-fluid model proposing:

- $\tau_i$ controlled by the light (gas) phase (LPD), giving $f_i = f_g$ and $\rho_i = \rho_g$.
- $\tau_i$ controlled by the heavy (liquid) phase (HPD), giving $f_i = f_l$ and $\rho_i = \rho_l$.
- $\tau_i$ controlled by the ratio of the absolute phase velocities, giving $f_i = f_g$ and $\rho_i = \rho_g$ for $|U_g| > |U_l|$, and $f_i = f_l$ and $\rho_i = \rho_l$ for $|U_g| < |U_l|$.

Here we assume that $\tau_i$ is controlled by the light (gas) phase for counter-current flows, while the other two approaches are tested in Section 3.1.2 in terms of multiple solutions boundaries.

Considering Eq. (2b) equal to zero ($F = 0$), the fully developed steady-state solution for the liquid level is obtained; all the geometrical relations needed for the computations are reported in Taitel and Dukler (1976). As shown by Landman (1991), Barnea and Taitel (1992), and Brauner (2002), multiple hold-up solutions can occur for concurrent and counter-current inclined pipexbrk flows.

The study of multiple solutions is usually carried out considering dimensionless variables, see for example Landman (1991); the Lockhart and Martinelli $X^2$ and the inclination parameter $Y$ are introduced for stratified flows by Taitel and Dukler (1976) giving