International Journal of Thermal Sciences 128 (2018) 84–93

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Predictive analytical expression of the Nusselt number for mixed convection in a lid-driven cavity filled with a stable-stratified fluid

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A R T I C L E I N F O

Keywords:
Lid-driven cavity
Mixed convection
Natural convection
Forced convection
Scale analysis
Numerical simulation

A B S T R A C T

In this work, a scale analysis is applied over the two-dimensional steady flow of stable-stratified fluid in a lid-driven square cavity. Scales for the viscous and thermal boundary layers are pursuit, and adjusted with numerical results. The flow parameters investigated covers the range $10^3 \leq Gr < 10^6$, $100 \leq Re \leq 2500$, and $10^{-1} \leq Pr \leq 10^2$. The outcome of the scale analysis is an expression for the surface-averaged Nusselt number which is capable of predicting the heat transfer within a 10% error range.

1. Introduction

Usually, mixed convection is referred due to the simultaneous occurrence of forced and free convection. However, this classification does not reflect the truly behavior of the flow, as if forced convection might be disrupted by buoyancy in a sense of flow restrain. In fact, such conditions characterize a stable stratified flow that is common in nature due to the solute or temperature gradients in the ocean and in the atmosphere [1].

In this work, it is considered the lid-driven flow inside a square cavity subjected to a gravitational stable condition where the buoyant-induced flow does not occur spontaneously. Alternatively, the flow is established by the movement of the cavity top surface, and buoyancy acts as a hindrance factor. A criteria for the absence of free convection depends on the magnitude and orientation of the thermal gradient [2] which is given as:

$$\frac{dT}{dy} > -\frac{gT}{c_p V}\left(\frac{\partial V}{\partial T}\right)_p \tag{1}$$

being c_p the constant pressure specific heat, g the gravity, p the pressure, T the temperature and V the fluid volume. Therefore, the free convection occurs whenever the temperature decreases with the increasing of the height and the magnitude of the gradient satisfies the inequality in Equation (1).

The lid-driven flow works out as a geometrical simplification for a number of complex nature problems such as in ocean dynamics analysis [3] and also in meteorological purposes [4]. Moreover, it is an approach for tackling engineering problems dealing with vapor deposition processes, green house covering, polymer processing [5], spray and flash drying, combustion of atomized fuels, cyclone evaporation, drying, dehydration [6], solar collectors, electronic devices cooling, lubrication technologies [7], nuclear reactor design [8], metal coating [9] and also in recirculating flows in the chemicals and food industry [10].

A fistful of studies for this problem are available considering a wide spectrum of applications. Initially, the lid-driven flow characteristics were studied by Ref. [11]. Although the main motivation had been to accomplish numerical validation of the multigrid method, their results for an isothermal flow have still been widely employed in numerical methods for benchmarking. An experiment to reproduce the geometry of the enclosure and evaluate the validity of flow assumptions such as two-dimensional and steady flow was built by Ref. [12]. It was observed that for Reynolds number, Re, higher than 3200, the flow exhibits three-dimensional characteristics and a time-dependent oscillating behavior when $Re > 6000$. More recently, numerical results for the isothermal flow were produced by a 1024×1024 mesh via the finite volume method [13]. Also, the flow of shear thinning [14], viscoplastic [15,16] and viscoelastic fluids [17] in lid-driven cavities were studied to describe the flow of molten polymers and food products.

Convection in a lid-driven cavity was empirically investigated by Ref. [18] as an addendum to their previous works. The influence of the cavity aspect ratio, the sliding-lid velocity and the thermal gradient over the heat transport was investigated. It was found that, for the parameters range considered, the heat transfer is independent of the cavity height and correlates well with the Grashof number according to the scale of $Gr^{1/3}$. The effects of the Prandtl number, Pr, over the

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https://doi.org/10.1016/j.ijthermalsci.2018.02.010
Received 31 August 2016; Received in revised form 8 February 2018; Accepted 8 February 2018
1290-0729/

convection heat transfer in a buoyant-shear-induced flow were numerically studied by Ref. [19]. Such effect over the flow behavior was more noticeable for predominantly buoyant-induced flows (i.e. Richardson number higher than unity, $Ri > 1$). A horizontally-heated square lid-driven cavity was numerically investigated by Ref. [20] where the effects of Gr, Re and Pr were accounted.

A gravitational-stable condition was considered upon a cavity with adiabatic side walls and a sinusoidal velocity at the top surface [21] as well as a constant velocity [22]. In the latter the numerical results have shown that an increase in the lid velocity intensifies the flow circulation and, therefore, enhances heat transfer. Conversely, an increase in buoyant forces might lead to a diffusive heat transfer regime due to the fluid stagnation in the cavity lower hemisphere. The stable-stratified three dimensional flow was numerically simulated by Refs. [1] and [23]. The mixed convection in a horizontally-heated two-sided-driven cavity was investigated by Ref. [24] where three boundary conditions resulting in shear-aided and shear-opposed flow were considered. Subsequently [25], incorporated the three-dimensional aspects into the modelling of the two-sided-driven cavity. The time-related characteristic of the flow were studied by Ref. [6] who solved numerically the unsteady balance equations. The effects of inclination in a gravitational stable enclosure were numerically investigated by Ref. [26], whereas the combined heat and mass transport was studied by Ref. [27].

In fact, the temperature gradient orientation in a lid-driven cavity under the action of buoyancy acts over the flow noticeably as it was studied by Refs. [28] and [29]. The influence of the thermal boundary conditions were investigated by Ref. [5] who solved the balance equations via Galerkin finite volume method. The problem boundary conditions varied from a non-uniform heated bottom with cold side walls to a uniformly heated bottom wall and linearly heated side walls. The occurrence of Hopf bifurcation upon the simultaneous variations of Gr and Re was studied by Ref. [30]. The heat transfer in a lid-driven cavity whose base is subjected to a uniform heat flux were numerically simulated either for two-dimensional and steady flow [31] and also to three-dimensional and unsteady flow [32].

Remarkably, none correlation to predict the boundary layer thickness and the surface-averaged Nusselt number Nu_{av} in a lid-driven square cavity subjected to a gravitational stable condition was found in literature. Therefore, a phenomenological discussion based on the influence of buoyancy over the heat transfer is presented here. A scale analysis is employed to pursuit an analytical correlation for the Nu_{av} as a function of Gr, Re and Pr. Subsequently, the analytical expressions are compared with numerical results for the sake of verification. The scale analysis is a problem-solving method initially formulated by Ref. [33] and then applied to analyze a variety of problems regarding convection heat transfer, for instance. Recently, the method was employed to study the free convection on a vertical surface with time-dependent boundary conditions [34–36].

2. Mathematical modeling

In the schematic representation displayed in Fig. 1, the cavity top surface is kept at a constant temperature T_h higher than the temperature at the bottom T_c. The side walls are adiabatic. A Cartesian coordinate system is placed at the cavity lower left corner with u and v standing for the velocity components in x and y-directions, respectively. The cavity top surface slides with constant velocity U_H.

The flow is assumed as steady, laminar, two-dimensional and with constant properties, which implies Newtonian and incompressible fluid. Radiative heat transfer is not accounted in the calculations. The conservation equations of mass, momentum (x and y-directions) and energy are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

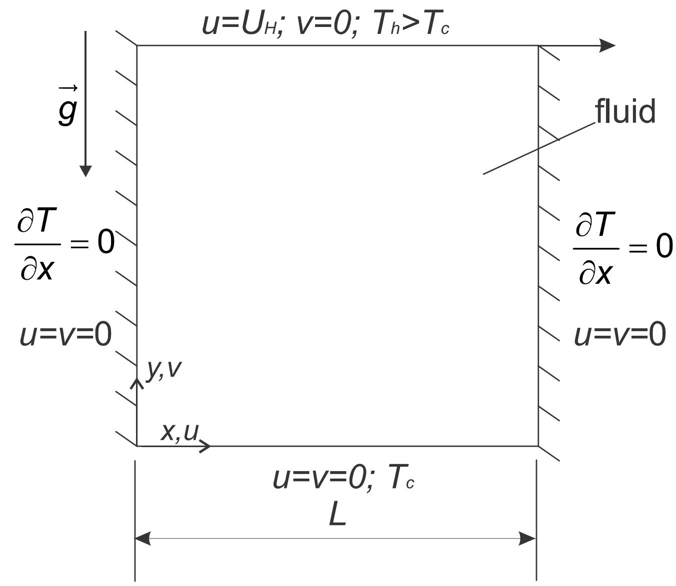

Fig. 1. Problem geometry and boundary conditions.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_c) \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{5}$$

where p is the pressure, ρ is the specific mass, υ represents the cinematic viscosity, g is the gravity, and β, T and α are, respectively, the isobaric volume expansion coefficient, the temperature and the thermal diffusivity. The flow is assumed incompressible and the effects of buoyancy are accounted in Equation (4) by the Boussinesq-Oberbeck approximation [37]. According to [38] the employment of such approximation is possible only for the cases where the influence of temperature and pressure causes a variation of less than 10% in the fluid properties.

The local heat transfer coefficient h over the cavity bottom surface is defined as:

$$h = \frac{k}{T_c - T_h}\frac{\partial T}{\partial y}\bigg|_{y=L} \tag{6}$$

being h_{av} the average heat transfer coefficient, as follow.

$$h_{av} = \frac{1}{L}\int_0^L h\, dx \tag{7}$$

Thus, the local and the surface-averaged Nusselt number are:

$$Nu = \frac{hL}{k} \tag{8}$$

$$Nu_{av} = \frac{h_{av}L}{k} \tag{9}$$

with Nu_{av} regarding analytical results and Nu_{av}^* numerical ones.

The stream function Ψ is defined as follows.

$$\Psi = \int_0^L u dy = \int_0^L -v dx \tag{10}$$