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Analytical solution of Fourier and non-Fourier heat transfer in longitudinal fin with internal heat generation and periodic boundary condition

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ABSTRACT

In this paper, the analytical solution of Fourier and non-Fourier model of heat transfer in longitudinal fin in presence of internal heat generation has been studied under periodic boundary condition. The whole analysis is given in dimensionless form. These two mathematical models have been solved analytically, using Laplace transform technique. Temperature distribution in longitudinal fin is measured using residual theorem in complex plane for the inverse Laplace transform technique. Thermal wave nature is appeared for small value of F_o. The longitudinal fin temperature is evaluated for different value of parameters with respect to space coordinate. The effect of variability of different parameters on temperature distribution in fin is studied in detailed. It has been observed that the cooling process is faster in non-Fourier model in comparison to Fourier model.

1. Introduction

In mechanical process heat is generated in the machines. How to release this generated heat in the environment this is a big problem. Heat transfer rate can be increased by increasing (i) the heat transfer coefficient (ii) the temperature difference between object and environment and (iii) the surface area. Heat transfer coefficient stretched up to maximum limit and second option is not in our hand. Now in third option, we can extend the area of the surface which is known as extended surface or fin. Extended surfaces are widely used in industry such as auto-mobile industry, chemical plants, food, power plants, air conditioning, refineries, medical science, petroleum, refrigeration, chilling plant, cold storage, electrical devices etc. Thermal conductivity of fin material should be high so that heat can be easily transfer from one place to another place.

In many engineering application during conduction, internal heat is generated in the system. For example heat is generated in electric wire when electricity flow form it. In nuclear reactors, heat is generated due to neutron absorption. In unsteady state condition, application of fins are found in many industrial applications such as cylinders of air cooled aircraft, Jet engines, electronic items etc. In different type heat problems, fins play an important role to dissipate heat in the environment. Shape of the fins and periodic type boundary conditions are very useful in industrial applications. There are many examples of periodic conduction such as atmospheric temperature cycle, heat transfer from internal combustion engines.

Fourier's law is appropriate for describing the conductive heat transfer in many engineering problems. In the field of extended surfaces, many mathematical models of heat conduction exist in the literature. Cattaneo (1958) [1] and Vernotte (1958) [2] suggested independently a hyperbolic heat conduction model with a finite propagation speed. This is known as the non-Fourier model and defined by following equation

$$q(x, t) + \tau \frac{\partial q(x, t)}{\partial t} = -k\nabla T(x, t). \quad (1)$$

Myers (1971) [3] studied periodic heat conduction in longitudinal fin of rectangular profile and solve it using the method of complex combination. Yang (1972) [4] studied periodic heat transfer in straight fin, effect of different parameters on temperature was studied in detail. Aziz and Na (1981) [5] studied periodic heat transfer in convecting fin with temperature dependent thermal conductivity and coordinate dependent heat transfer coefficient. They have used the perturbation analysis method in solution. Aziz and Na (1981a) [6] studied longitudinal fin of rectangular profile with uniform thickness and periodic base temperature. The base temperature is considered periodic and it oscillates around mean value of the temperature. They used the perturbation method in solution. Eslinger and Chung (1979) [7] and Houghton et al. (1992) [8] studied the steady state periodic heat transfer in convective fin. Aziz and Lunardini (1994) [9] considered periodic heat transfer process in extended surfaces. One and two dimensional heat transfer models are considered in their study and effect

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Nomenclature

Ac	cross section area of fin (m^2)
T_b	fin temperature (K)
T_∞	sink temperature (K)
$T_{b,m}$	base temperature (K)
ρ	density of fin material (kgm^{-3})
c	specific heat ($Jkg^{-1}K^{-1}$)
k	thermal conductivity ($Wm^{-1}K^{-1}$)
q^*	internal heat generation (Wm^{-2})
q^*_∞	internal heat generation at sink temperature (Wm^{-2})
ε	internal heat generation parameter (K^{-1})
h	heat transfer coefficient ($Wm^{-2}K^{-1}$)
P	fin perimeter (m)
x	spatial coordinate (m)
L	length of fin (m)
ω	periodicity (s^{-1})
t	time (s)
τ	relaxation time (s)

Dimensionless variables

θ	fin temperature
X	coordinate space
F_0	Fourier number or dimensionless time
N	fin parameter
G	generation number
ε_G	internal heat generation
Ω	dimensionless periodicity
η	relaxation time
A	amplitude of the input temperature

Abbreviations

PHTE	Parabolic Heat Transfer Equation
HHTE	Hyperbolic Heat Transfer Equation

of different parameters studied in the model.

Chen and Lin (1993) [10] studied hyperbolic heat conduction problem using Laplace transform and control volume methods in the solution. Time dependent terms are removed by Laplace transform method and then the transformed equations are discretized by the control volume method. Lin (1998) [11] studied the effect of thermal relaxation time i.e. non-Fourier effect on the convective fin with periodic boundary conditions. In this solution, hybrid numerical method combining the Laplace transform and control volume methods was used. Bishri (1999) [12] studied the non-Fourier heat conduction in a finite medium which is insulated from both sides with periodic boundary conditions. The finite integral transform method was used in the solution of problem. Tang and Arakin (1999) [13] studied the transient heat conduction in finite rigid slab. They used Greens function method and finite integral transform technique in the solution.

Kraus et al. (2001) [14] wrote a book on extended surface heat transfer which covers each and every topic, chapter 16 is devoted to study of transient heat transfer in extended surfaces. Chapter 17 is devoted for study of periodic heat flow in fins. Cossali (2004) [15] studied non-Fourier heat conduction model for a one dimensional slab considering periodic boundary condition in material. They used the transfer function technique in solution. Yang (2005) [16] studied heat conduction in non-Fourier fin problem with periodic boundary conditions, Finite difference method and modified Newton-Raphson method applied in the solution. Li (2010) [17] studied the heat transfer in irregular shape of fins. Time dependent, two dimensional heat conduction problem is considered and finite element method was used in the solution. The coupled conductive, convective and radiative heat transfer in moving porous fins of trapezoidal, convex and concave parabolic profiles have been studied by Ma et al. (2017) [18]. They solved the problem by using spectral element method. The spectral Collocation method is used for solution of transient thermal process in the moving plate with temperature dependent properties and heat generation. In this paper, they observed high rate accuracy for this problem [19].

Mosayebidorcheh et al. (2015) [20] discussed about optimum design for different type shapes of fin. They studied temperature dependent heat generation problem and used least square method in the solution. Nonlinear temperature distribution equation in solid and porous longitudinal fin with temperature dependent internal heat generation has been solved using differential transform method [21]. They found that this method can achieve more suitable result in comparison to numerical methods. The analytical solution for efficiency of semi-spherical porous fin under fully wet conditions by using temperature and humidity ratio differences as the driving forces for heat and mass transfer mechanism has been studied by Hatami et al. (2014) [22]. Hatami et al. (2014, 2014, 2015) [23–25] studied experimental analysis of the optimized finned-tube heat exchanger for OM314 diesel exhaust exergy recovery; and two cases of heat exchangers (HEXs) which previously were used in exhaust of internal combustion engines (ICEs) are modeled numerically to recover the exhaust waste heat ([24]). They observed that maximum exergy recovered in high engine loads. Hatami (2016) [26] studied about optimal profile of wavy-wall for an enclosure to study the nanoparticles treatment around the heated cylinder. He used finite element method and response surface method in the solution. Hatami and Jing (2017) [27] studied about wavy direct absorber solar collector using aluminium oxide as nanofluid and problem solved by response surface method. Fakour et al. (2015) [28] have investigated the micropolar fluid flow in a channel subjected to chemical reaction analytically using least square method; and also compared the results obtained from proposed method with numerical method. Tang et al. (2017) [29] studied quarter circular encloser with two sinusoidal wavy walls and two straight walls.

Das and Prasad (2015) [30] have analysed the simultaneous inverse prediction of two parameters, porosity and thermal diffusivity of the fluid in porous fin. The prediction of parameters is studied in an annular hyperbolic fin with temperature dependent thermal conductivity in steady state [31]. The Adomian decomposition method has been used for solution of convective and radiative moving fin having variable thermal conductivity which is studied by Singla and Das (2014) [32], this type problem for variable thermal conductivity also studied by Singh et al. (2013, 2015) [33,34]. Singh et al. (2015) [35] studied the thermal investigation of porous media stapped fin made from different ceramic porous medium having temperature dependent internal heat generation. Singh et al. (2014) [36] studied about convective-radiative fin with temperature dependent thermal conductivity, heat transfer coefficient and wavelength dependent surface emissivity. Pourmehran et al. (2015) [37] studied about thermal and flow analysis of a fin shaped microchannel heat sink cold by different nano fluids based on saturated porous medium.

Different type fins such as longitudinal, annular and spine operating in dehumidifying conditions have been studied by Kundu (2008) [38]. Kundu and Lee (2013) [39] have studied the temperature response in fin with internal heat generation due to heat conduction describe by Fourier and non-Fourier laws. They solved their problem using analytical method. Ahmadikia and Rismanian (2011) [40] obtained analytical solution of Fourier and non-Fourier heat conduction model with periodic boundary conditions. They used the Laplace transform method in the solution and studied the effect of different parameters on temperature in fin. Aziz et al. (2012) [41] studied unknown base