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# Effect of particle stress tensor in simulations of dense gas–particle flows in fluidized beds

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ABSTRACT

A two-fluid model based on the kinetic theory of granular flow for the rapid-flow regime and the Coulomb friction law for the quasi-static regime is applied to predict the hydrodynamics of dense gas–particle flow in a three-dimensional fluidized bed. Two different models for the particle stress tensor that use different constitutive equations in the elastic-inertial regime are examined to assess their ability to predict bed dynamics. To understand how particle stress models affect structural features of the flow, a quantitative analysis is performed on some important aspects of the mechanics of bubbling beds that have received relatively little attention in the literature. Accordingly, different flow regimes are identified in the context of fluidized beds through the dimensionless inertial number, and the main characteristics of each regime are discussed. In addition, how the particle stress tensor manifests itself in the bubble characteristics, natural frequency of the bed, and particle Reynolds stress are investigated, all of which help to better understand the complex dynamics of the fluidized bed. The numerical results are validated against published experimental data and demonstrate the significant role of the stress tensor in the elastic-inertial regime.



## Introduction

Successful predictions of the hydrodynamics of fluidized beds require accurate modeling of particulate stress terms in the momentum equation. It is generally accepted to apply the kinetic theory of granular flow to model the kinetic-collisional stresses in dilute regions, corresponding to the rapid-flow regime, where the streaming of individual particles and the binary collisions between them are the dominant mechanisms of momentum transport; see e.g., Lun, Savage, Jeffrey, and Chepurniy (1984), Savage (1984), and Van Wachem, Schouten, Van den Bleek, Krishna, and Sinclair (2001). However, there is no general consensus in modeling the frictional stresses in dense regions where there is sustained contact between particles. In their model, Johnson and Jackson (1987) assumed that the total stresses acting on the particle phase are the sum of the kinetic stresses and the frictional stresses. Although the validity of this assumption is not assured, it is capable of capturing the two extrema of granular flows, i.e., viscous flow and plastic flow (Srivastava & Sundaresan, 2003). Syamlal, Rogers, and O'Brien (1993) proposed a model, referred to as the Schaeffer model, in which the effects of the frictional stresses are activated only at particle volume fractions higher than $\varepsilon_p^{max}$, corresponding to the quasi-static slow-flow regime. For frictional stresses, following Schaeffer (1987), they assumed that the shear stress is proportional to the normal stress. In this model, there is a sharp transition between the rapid-flow and quasi-static regimes that could result in erroneous bubble shape and bed expansion (Makkawi, Wright, & Ocone, 2006). Using the additive approach of Johnson and Jackson (1987), Srivastava and Sundaresan (2003) proposed another model for particulate phase stresses, referred to as the Princeton model. In this model, the frictional stress is added to the kinetic-collisional stress tensor in the intermediate regime, referred to as the elastic-inertial regime (Campbell, 2002). In this model, the frictional stresses affect the granular flow at a minimum frictional particle volume fraction $\varepsilon_p^{min}$.

Although there are other frictional stress models available, e.g., see Schneiderbauer, Aigner, and Pirker (2012), the most pervasive ones for the simulation of bubbling beds are the Schaeffer and Princeton models. Therefore, it is relevant to determine which of these two models is more realistic, and this in turn requires a detailed knowledge of how these models affect the simulated flow

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structure of the gas–particle flows. These models have already been the subject of several studies that assessed their ability to predict the overall behavior of fluidized beds, e.g., see Benyahia (2008), Passalacqua and Marmo (2009), Reuge et al. (2008), and Verma, Deen, Padding, and Kuipers (2013). Passalacqua and Marmo (2009) performed a comparison of the frictional stress models applied to a two-dimensional (2D) fluidized bed and used the mean bubble diameter to interpret their numerical results. Reuge et al. (2008) studied the effects of dissipation parameters on the dynamics of the bed with a focus on the bed expansion ratio and its fluctuation. Although these are undoubtedly useful parameters in the study of fluidized beds, the flow patterns were not discussed. In a more comprehensive study, Verma et al. (2013) investigated the effect of several parameters on the numerical simulation of a three-dimensional (3D) bubbling fluidized bed and demonstrated the significance of the frictional models. Most of studies have only focused on a comparison of the model results. Relatively few studies have investigated how the particle stress models affect structural features such as flow regimes, flow patterns, velocity profiles, and bubble formation; such studies would advance our understanding of the complex behavior of these multiphase flows.

This paper reports a more in-depth study of two different models for the particle stress tensor in the elastic-inertial regime and assesses their ability to predict the hydrodynamics of a 3D cylindrical fluidized bed. A major objective is to gain insight into how these models modify the simulated flow structure. The paper describes quantitatively some important features of the mechanics of the bubbling/slugging beds that have received relatively little attention in the literature. To that end, different flow regimes are identified, in the context of fluidized beds, through the dimensionless inertial number, and the main characteristics of each regime are discussed. To the best of our knowledge, it is the first time that contours of inertial number are used to visualize the flow properties. Analysis of the flow properties for a range of gas–particle regimes based on their inertial number enhances our understanding of the flow behavior in such a complex multiphase system. In addition, the effect of the particle stress tensor on bubble formation, bubble behavior, natural frequency of the bed, and particle Reynolds stress are investigated. The numerical code Multiphase Flow with Interphase eXchanges (MFIX) is used to perform the simulations (Syamlal et al., 1993), using the Eulerian–Eulerian framework. The results are validated against experimental data presented in Laverman et al. (2012).

## Mathematical modeling

The Eulerian–Eulerian two-fluid model, based on the locally averaged equations derived by Anderson and Jackson (1967), is used to simulate an isothermal gas–particle system.

The governing equations, for phases $m$, $m'$ (where $m$, $m'$ = g, p for gas or particles, respectively, and $m \neq m'$), are conservation of mass

$$\frac{\partial(\varepsilon_m\rho_m)}{\partial t}+\frac{\partial(\varepsilon_m\rho_m u_{mi})}{\partial x_i}=0, \tag{1}$$

and conservation of momentum

$$\frac{\partial(\varepsilon_m\rho_m u_{mi})}{\partial t}+\frac{\partial\left(\varepsilon_m\rho_m u_{mj}u_{mi}\right)}{\partial x_j}=-\varepsilon_m\frac{\partial P_{\rm g}}{\partial x_i}+\frac{\partial\tau_{mij}}{\partial x_j}$$
$$-\gamma_m\frac{\partial P_{\rm p}}{\partial x_i}+\beta(u_{m'i}-u_{mi})+\varepsilon_m\rho_m g_i, \tag{2}$$

where $\gamma_m = 1$ if and only if $m$ = p; otherwise, $\gamma_m = 0$. The gas phase (air) is modeled as an ideal gas so that $\rho_{\rm g}$ is calculated from $P_{\rm g}$. Here $\rho$, $\varepsilon$, $\mathbf{u}$, $P$, $\bar{\bar{\tau}}$, $\beta$, and (non-subscript) $g$ represent the density, volume fraction, velocity vector, pressure, stress tensor, inter-phase momentum transfer coefficient, and gravitational acceleration, respectively. Noting that $\varepsilon_{\rm p}+\varepsilon_{\rm g}=1$, only $\varepsilon_{\rm p}$ is treated as an independent variable. The interfacial drag coefficient $\beta$ is defined by

$$\beta=18\varepsilon_{\rm g}\varepsilon_{\rm p}\mu\frac{F\left(\varepsilon_{\rm p},Re\right)}{d_{\rm p}^2}, \tag{3}$$

where $F\left(\varepsilon_{\rm p},Re\right)$ and $d_{\rm p}$ are the dimensionless drag force and particle diameter, respectively, and $\mu$ is the gas viscosity. In this study, the drag force proposed by Beetstra, Van der Hoef, and Kuipers (2007) is used; it is defined as

$$F\left(\varepsilon_{\rm p},Re\right)=\frac{10\varepsilon_{\rm p}}{(1-\varepsilon_{\rm p})^2}+(1-\varepsilon_{\rm p})^2\left(1+1.5\sqrt{\varepsilon_{\rm p}}\right)$$
$$+\frac{0.413Re}{24(1-\varepsilon_{\rm p})^2}\left[\frac{(1-\varepsilon_{\rm p})^{-1}+3\varepsilon_{\rm p}(1-\varepsilon_{\rm p})+8.4Re^{-0.343}}{1+10^{3\varepsilon_{\rm p}}Re^{-(1+4\varepsilon_{\rm p})/2}}\right], \tag{4}$$

where $Re = d_{\rm p}|\mathbf{u}_{\rm g}-\mathbf{u}_{\rm p}|\rho_{\rm g}\varepsilon_{\rm g}/\mu$ represents the particle Reynolds number.

To close Eq. (2) for the particle phase, an expression for the stress tensor is required. Following Johnson and Jackson (1987), the particle stress tensor is assumed to be the sum of the kinetic-collisional stresses and the frictional stresses; i.e.,

$$\tau_{{\rm p}ij}=\tau_{{\rm p}ij}^{\rm kc}+\tau_{{\rm p}ij}^{\rm f}, \tag{5}$$

where the kinetic-collisional stress tensor, commonly modeled by the kinetic theory of granular flow, is given by

$$\tau_{{\rm p}ij}^{\rm kc}=\left(\eta\mu_{\rm b}\frac{\partial u_{{\rm p}i}}{\partial x_i}\right)\delta_{ij}+2\mu_{\rm p}S_{{\rm p}ij}, \tag{6}$$

and the frictional stress tensor is given by

$$\tau_{{\rm p}ij}^{\rm f}=2\mu_{\rm f}S_{{\rm p}ij}, \tag{7}$$

where $\delta_{ij}$ is the Kronecker delta,

$$\delta_{ij}=\begin{cases}1, & i=j,\\ 0, & i\neq j,\end{cases} \tag{8}$$

and the quantity $\eta$ is defined by

$$\eta=\frac{1+e}{2}, \tag{9}$$

where $e$ is the particle–particle coefficient of restitution. In this study, the value of $e$ is 0.86.

The bulk viscosity is given by Lun et al. (1984),

$$\mu_{\rm b}=\frac{256}{5\pi}\mu'\varepsilon_{\rm p}^2 g_0, \tag{10}$$

where

$$\mu'=\frac{5}{96}\rho_{\rm p}d_{\rm p}\sqrt{\pi\Theta_{\rm p}}, \tag{11}$$

and $g_0$ is the radial distribution function taking into account the probability of particle collision.

The particle strain-rate tensor is given by

$$S_{{\rm p}ij}=\frac{1}{2}\left(\frac{\partial u_{{\rm p}i}}{\partial x_j}+\frac{\partial u_{{\rm p}j}}{\partial x_i}\right)-\frac{1}{3}\frac{\partial u_{{\rm p}i}}{\partial x_i}. \tag{12}$$

Lun et al. (1984) also proposed an expression for the particle viscosity. However, they did not consider the effect of the interstitial fluid. Following Ma and Ahmadi (1988), the particle viscosity, in which the interstitial fluid effect is included, is given by

$$\mu_{\rm p}=\left(\frac{2+\alpha}{3}\right)\left[\frac{\mu_{\rm p}^*}{g_0\eta(2-\eta)}\left(1+\frac{8}{5}\eta\varepsilon_{\rm p}g_0\right)\left(1+\frac{8}{5}\eta(3\eta-2)\varepsilon_{\rm p}g_0\right)+\frac{3}{5}\eta\mu_{\rm b}\right], \tag{13}$$