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Cubature H_∞ information filter and its extensions

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ABSTRACT

State estimation for nonlinear systems with Gaussian or non-Gaussian noises, and with single and multiple sensors, is presented. The key purpose is to propose a derivative free estimator using concepts from the information filter, the H_∞ filter, and the cubature Kalman filter (CKF). The proposed estimator is called the cubature H_∞ information filter (CH_∞IF); it has the capability to deal with highly nonlinear systems like the CKF, like the H_∞ filter it can estimate states with stochastic or deterministic noises, and similar to the information filter it can be easily extended to handle measurements from multiple sensors. A numerically stable square-root CH_∞IF is developed and extended to multiple sensors. The CH_∞IF is implemented to estimate the states of a nonlinear permanent magnet synchronous motor model. Comparisons are made with an extended H_∞ information filter.

1. Introduction

State estimation for nonlinear systems is an active area of research and is essential for many real-life applications. One of the most preferred estimators for nonlinear systems is the extended Kalman filter (EKF), which is an extended version of the classical Kalman filter [2,36]. Other notable nonlinear state estimators include Gaussian-mixture filters [18], quadrature filters [3], Gaussian–Hermite filters [5], Fourier–Hermite filters [32], sliding mode observers [13], central difference filters [27], particle filters [25,6,12], unscented Kalman filters (UKFs) [20,21] and cubature Kalman filters (CKFs) [4].

The EKF formulation is based on the first order Taylor's series approximation of nonlinear state and measurement models (Jacobians), and may not be suitable for *highly* nonlinear systems. For some models, such as piecewise continuous nonlinear systems [32], where it is difficult to obtain the Jacobians, derivative filters like EKF should be avoided. Furthermore, a priori statistical knowledge of process and sensor noise is required for EKF. Deterministic sigma-point filters like UKFs and CKFs, "or particle filters" can be used to estimate the states of the nonlinear system without evaluating the Jacobians. But UKF and CKF have limited capability to deal with non-Gaussian noises. Particle filters can handle non-Gaussian noises, but their performance is dictated by the number of stochastically selected samples or particles. For better accuracy more particles are required and hence they are computationally expensive filters. More recently, H_∞ filters and their variants [14,7,33,34,1,37,11,30,28,29] have been investigated and utilised to deal with non-Gaussian noises. An extended H_∞ filter ($EH_\infty F$) can be used for nonlinear systems with non-Gaussian noise, but they need Jacobians. The CKF and H_∞ filters are combined to handle nonlinear systems with unknown noise statistics [10]; however this estimator cannot directly deal with measurements from multiple sensors. In several real life applications, where the measurements come from different sets of sensors, Kalman filters are seldom used. Alternatively, an algebraic equivalent form of Kalman filter, an information filter is preferred over the standard Kalman filter due to its simpler update stage. For nonlinear state estimation with multiple sensors, an extended information filter (EIF) can be used [26]. However, the EIF is not a derivative free filter and requires Jacobians during the prediction and update stages and hence is not preferred for highly nonlinear systems; and they have limited capability to handle non-Gaussian noise. A few derivative free information filters like unscented information filters [24], cubature information filters [9,19], etc. have been recently proposed for nonlinear systems with

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Gaussian noises. H_∞ filters in the information domain have been extended to nonlinear systems, but many of these EH_∞F s are not suitable for nonlinear systems where the nonlinearity is severe; this is due to the fact that EH_∞F s are Jacobian based filters. In [8], we presented an earlier version of this work consisting of basic cubature H_∞ information filter. In this paper, we present the cubature H_∞ information filter (CH_∞IF) and its extensions, which have the capability to estimate the states of highly nonlinear systems in the presence of Gaussian or non-Gaussian noises, and can handle measurements from multiple sensors.

The paper is structured as follows. Filtering preliminaries are given in Section 2; the CH_∞IF is derived in Section 3; the square-root extension of the CH_∞IF is presented in Section 4; the applicability of multi-sensor CH_∞IF for state estimation of a permanent magnet synchronous motor is described in Section 5; and conclusions are given in Section 6.

2. Filtering preliminaries

This section presents the most relevant filtering approaches required for development of the CH_∞IF. The key focus will be on the EH_∞F, the EIF and the CKF. Note that these filters will only be briefly discussed here; for more details see for example [36] and [26] for EH_∞F s and EIFs, respectively, and [4] for the CKF.

2.1. Extended H_∞ filter

In the last two decades there has been an increasing interest in robust filters using H_∞ theory and several authors have come up with different forms of the so-called H_∞ filters [14,7,33,39]. In this section a game theory based H_∞ filter will be discussed which is mainly based on [36,7,40].

The nonlinear discrete plant and measurement models are given by

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k \tag{2}$$

where the state vector, control input and the measured outputs are denoted by $\mathbf{x}_k$, $\mathbf{u}_k$ and $\mathbf{z}_k$, respectively. The functions $\mathbf{f}$ and $\mathbf{h}$ are the nonlinear functions of states and control inputs. The plant and measurement noises are represented by $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$. In most of the Kalman filtering approaches these noises are assumed as Gaussian and have zero-mean, whereas in the H_∞ filtering approaches they are not assumed to follow any particular probability distribution. In this section, it will be assumed that $\mathbf{w}_k$ and $\mathbf{v}_k$ can vary randomly or they can be deterministic, and they can also have non-zero mean.

The cost function for the H_∞ filter is of the form [36,7],

$$\mathbf{J}_\infty = \frac{\sum_{k=0}^{N-1} \| \mathbf{x}_k - \hat{\mathbf{x}}_k \|^2_{\mathbf{L}_k}}{\| \mathbf{x}_0 - \hat{\mathbf{x}}_0 \|^2_{\mathbf{P}_0^{-1}} + \sum_{k=0}^{N} \left(\| \mathbf{w}_k \|^2_{\mathbf{Q}_k^{-1}} + \| \mathbf{v}_k \|^2_{\mathbf{R}_k^{-1}} \right)} \tag{3}$$

where the weighting matrices $\mathbf{P}_0$, $\mathbf{Q}_k$, $\mathbf{R}_k$, and $\mathbf{L}_k$ are symmetric positive definite weighing matrices chosen by the user based on the problem at hand. Note that this cost function is slightly different from the one in [36,7]. The numerator of (3) is the norm of the state estimation errors, however if one has to estimate the linear combination of states then the numerator of (3) has to be an error norm of a linear combination of states as given in [36,7]; which can however be absorbed by $\mathbf{L}_k$ in (3).

In the worst case noise and the initial conditions, the aim of the H_∞ filter is to minimise the state estimation error in such a way that the performance measure J_∞ is bounded as

$$\sup \mathbf{J}_\infty < \gamma^2 \tag{4}$$

where 'sup' means supremum and the attenuation parameter $\gamma > 0$.

Several solutions to this H_∞ problem are available in [7,33,36], etc. However, in this paper the solution given in [40] will be used, as the H_∞ filter structure in [40] closely resembles with Kalman filter. For nonlinear systems, an EH_∞F can be used where the nonlinear functions are replaced by the Jacobians. Like EKF, an EH_∞F can be expressed in two stages (prediction and update) [40].

Prediction stage in the **E**$\mathbf{H}_\infty$***F***: The predicted state and predicted auxiliary matrix are:

$$\mathbf{x}_{k|k-1} = \mathbf{f}(\mathbf{x}_{k-1|k-1}, \mathbf{u}_{k-1}) \tag{5}$$

$$\mathbf{P}_{k|k-1} = \nabla\mathbf{f}\mathbf{P}_{k-1|k-1}\nabla\mathbf{f}^T + \mathbf{Q}_k, \tag{6}$$

where $\nabla\mathbf{f}$ is the Jacobian of $\mathbf{f}$ evaluated at $\mathbf{x}_{k-1|k-1}$.

Update stage in the ***E****$\boldsymbol{H}_\infty$****F****:* The updated state and updated auxiliary matrix are:

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{K}_\infty[\mathbf{z}_k - \mathbf{h}(\mathbf{x}_{k|k-1}, \mathbf{u}_k)] \tag{7}$$

$$\mathbf{P}^{-1}_{k|k} = \mathbf{P}^{-1}_{k|k-1} + \nabla\mathbf{h}^T\mathbf{R}_k^{-1}\nabla\mathbf{h} - \gamma^{-2}\mathbb{I}_n \tag{8}$$

where $\mathbb{I}_n$ is the nth order identity matrix, $\nabla\mathbf{h}$ is the Jacobian of $\mathbf{h}$ evaluated at $\mathbf{x}_{k|k-1}$, and

$$\mathbf{K}_\infty = \mathbf{P}_{k|k-1}\nabla\mathbf{h}^T[\nabla\mathbf{h}\mathbf{P}_{k|k-1}\nabla\mathbf{h}^T + \mathbf{R}_k]^{-1}. \tag{9}$$

2.2. Extended information filter

The Kalman filter propagates the state and covariance matrix at various stages. However, in some applications like multi-sensor state estimation, an information filter (an alternate form of Kalman filter), is preferred due to its simpler update stage to fuse the measurements