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Improving first order sliding mode control on second order mechanical systems

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ABSTRACT

In this paper is proposed a control algorithm based on the first order sliding mode technique. The control design adds an exponential reaching law and a disturbance estimator to improve performance, achieving a reduction of the convergence time to the reference, as well as a reduction of the reaching time towards the sliding surface. Also, by compensating the estimated disturbance, it is possible to reduce the amplitude of the chattering in the control signal. As the control design is intended to be applied in mechanical systems, a velocity observer design is also proposed. Bringing together the above aspects, the proposed controller renders an improved performance over the classical first order sliding mode controller. The stability of the closed-loop system is proved using quadratic functions. The performance of the proposed control structure is illustrated and compared with other controllers via numerical simulations and real-time experiments in a mechanical system.

1. Introduction

In many control problems' applications, there is a discrepancy between the actual plant and its mathematical representation used for control design purposes. These discrepancies arise from uncertainties, external perturbations, non-modeled friction, and non-well characterized plant parameters. Thus, designing a robust control algorithm capable of overcoming these discrepancies is a challenging task, some control techniques have proved to be useful to this purpose, like sliding mode control, since the main advantages of sliding mode technique are robustness, finite time convergence to the sliding surface, and reduced-order compensated dynamics [17].

Some previous works containing core ideas about sliding mode approach, such as the design of an exponential reaching law in order to reduce the convergence time to the sliding surface, are listed as follows. In [3], was proposed the reaching law method, which is complemented by a sliding mode equivalence technique. In [2,21], was proposed the design of a nonlinear reaching law by using an exponential function that dynamically adapts to the variations of the controlled system, and it was applied on multi-input/multi-output (MIMO) nonlinear systems. More recently, Yu and Sun [23] addressed a hierarchical sliding mode controller, based on an exponential reaching law, in order to achieve the set-point regulation of the longitudinal motion on a pendulum-driven spherical mobile robot.

In many control applications there is a need of an observer that converges to the system states in spite of the presence of unknown signals or uncertainty. This issue is addressed in [22], where is proposed a sliding-mode control approach for systems with mismatched uncertainties via a nonlinear disturbance observer. In [16] is proposed a control structure for a class of uncertain Lagrangian systems to solve the regulation and tracking control problems, the control structure is based on a discontinuous robust observer with the aid of a low-pass filter to estimate the perturbations affecting the plant. In [19] was proposed a new boundary layer sliding mode control design for chatter reduction using an uncertainty and disturbance estimator (UDE). In [4] a state and extended disturbance observer (DO) is designed for mismatched uncertain systems. In [17], sliding mode observers and differentiators are presented in a tutorial level. Concerning to the estimation of perturbations and disturbances, in [6] is designed a discontinuous disturbance observer to be used in feedback compensation of parameter uncertainties and exogenous disturbances.

Control of mechanical systems considering viscous friction can be found in [13,14], where are proposed sliding mode controllers, which include an H_∞ control on its sliding surface, to reduce unmatched perturbations on the unactuated link of the

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mechanical system. Moreover, in [12], by using the siliding mode technique, the regulation problem of a mechanical system with a position constraint, affected by Coulomb friction and an external perturbation is addressed. Also, Liu et al. [8] consider the problem of sliding mode control for a class of uncertain switched systems with parameter uncertainties and external disturbances.

In the present study is proposed for second order mechanical systems a sliding mode synthesis procedure including an exponential reaching law, a disturbance estimator, a discontinuous observer, and their stability proofs considering discontinuous friction, external perturbations, and non-well modeled parameters. The aforementioned synthesis constitutes the main contribution of the present work, which to the best of our knowledge had never been addressed before as a whole.

The study is organized as follows. The problem statement to solve the tracking problem in second order mechanical systems under external perturbations, and uncertainties, is presented in Section 2. The exponential reaching law and its convergence time analysis are presented in Section 3. In Section 4, is presented the velocity observer design along with the stability proof using a Lyapunov function. Section 5 presents a disturbance estimator given by a second-order low-pass Butterworth filter. In Section 6 is designed the control algorithm for the output feedback, and its stability is analyzed. Section 7 presents in simulation level a performance comparison of the proposed control approach against sliding mode controllers such as twisting algorithm, super twisting algorithm, and first order sliding mode control. Moreover, experiments were performed using a mass–spring–damper system using the proposed control approach. Finally, Section 8 presents some concluding remarks.

2. Problem statement

The aim of this paper is to propose a methodology to improve the performance of first order sliding mode control to solve the tracking problem in second order systems via output measurements. The system is considered to be under external perturbations and parametric uncertainties, in which both are considered bounded.

Consider the second order mechanical system represented by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}x_2\\ -ax_1-bx_2-f(x)+\tau+w\end{bmatrix} \tag{1}$$

$$y=x_1 \tag{2}$$

where x_1, $x_2\in\mathbb{R}$ are the position and velocity of the body, respectively, and only measurements of x_1 are available, a, b are known positive constants which are different from zero, $f(x)$ is a non-completely known nonlinear function, $\tau\in\mathbb{R}$ is the control input. To account for discrepancies in the model, a not completely known non-vanishing perturbation $w(t)$ is considered, which is upper bounded by a positive constant κ as

$$\sup_t|w(t)|\le\kappa. \tag{3}$$

For system (1) the following controller is proposed

$$\tau=\tilde{f}(x)+u \tag{4}$$

where $\tilde{f}(x)=f(x)+\Delta f(x)$, and Δf represents the error between f and $\tilde{f}$, which is considered upper bounded by a positive constant ζ. For a zero force input $\tau=0$, and zero disturbance ($w=0$), the system (1) has the following equilibrium point ($x_1=-f(x)/a, x_2=0$).

If it is desired that the equilibrium point be at the origin in steady state, then τ must be equal to the nonlinear function $f(x)$; otherwise if, it τ is a constant $\overline{\tau}$, the equilibrium point of interest can be considered as ($x_1=(\overline{\tau}-f(x))/a, x_2=0$).

3. Exponential reaching law

Let x_d be the reference trajectory, and let $\eta_1=x_1-x_d$ and $\eta_2=x_2-\dot{x}_d$ be the tracking errors, which are desired to be driven to zero. The first step in sliding mode control is to choose the switching surface s; in this case, in terms of the tracking errors. The typical choice of s, see [18], is

$$s=\eta_2+\lambda\eta_1,\quad\forall\lambda>0. \tag{5}$$

When the sliding surface is reached, the tracking errors η_1 and η_2 exponentially converge to zero. There are two stages in the sliding mode approach. The first stage, called reaching stage, is the step in which the errors η_1 and η_2 are attracted to the switching surface $s=0$. In the second stage, also known as sliding mode, the error vector "slides" on the surface until it reaches the equilibrium point $(0,0)$.

Having chosen the sliding surface, the next step would be to design the control law u that will allow the trajectories (η_1,η_2) to reach the sliding surface. To do so, the control law should be designed such that the following sliding condition is met

$$s\cdot\dot{s}<0,\quad\forall t. \tag{6}$$

To satisfy condition (6), $\dot{s}$ remains under external perturbations w and the bounded error Δf as follows:

$$\dot{s}=-\beta\cdot\mathrm{sign}(s)+w+\Delta f \tag{7}$$

with $\beta>\kappa+\zeta$ $\forall t$ condition (6) can be satisfied. Expression (7) is also called reaching law. The term $w(t)$ is a non-vanishing perturbation satisfying (3). It can be proved that the system trajectories reach the surface $s=0$, in finite time, using the following quadratic function:

$$V(s)=s^2 \tag{8}$$

and time-differentiating it along (6) to obtain

$$\dot{V}(s)\le-2(\beta-(\kappa+\zeta))|s|=-2(\beta-(\kappa+\zeta))\sqrt{V(s)}. \tag{9}$$

Integrating with respect to time the previous equation, and noting that $V(t)=0$, for $t\ge t_r$, one has that

$$t_0+\frac{\sqrt{V(t_0)}}{\beta-(\kappa+\zeta)}=t_r. \tag{10}$$

Hence, $V(t)$ converges to zero in finite time and, in consequence, a motion along the manifold $s=0$ occurs. Note that the reaching time can be reduced by increasing the value of β.

Now, consider the following exponential reaching law (see [3,23]) which is affected as well by perturbations and uncertainties

$$\dot{s}=-\frac{\beta}{N(s)}\cdot\mathrm{sign}(s)+w+\Delta f \tag{11}$$

where

$$N(s)=\delta_0+(1-\delta_0)e^{-\alpha|s|^p}>0,\quad\forall t \tag{12}$$

the constants $0<\delta_0<1$, $p>0$ and $\alpha>0$. Due to $N(s)$ is always strictly positive, the exponential reaching law given by (11) does not affect the stability of the reduced order system. From the exponential sliding surface stated in (11), one can see that if $|s|$ increases, $N(s)$ approaches δ_0, and therefore, $\beta/N(s)$ converges to β/δ_0, which is greater than β. This means that $\beta/N(s)$ increases as $|s|$ increases, and consequently, the convergence rate to $s=0$ will be faster. On the other hand, if $|s|$ decreases, then $N(s)$ approaches one, and $\beta/N(s)$ converges to β. This means that, when the systems' trajectories approach to the reference, $\beta/N(s)$ gradually decreases in order to reduce the control effort. By this way, the exponential reaching law allows the controller to dynamically adapt to the variations of s by letting $\beta/N(s)$ to vary between β and β/δ_0. If δ_0 is chosen to be equal one, the reaching law (11) becomes identical to the conventional reaching law $\dot{s}=-\beta\cdot\mathrm{sign}(s)+w+\Delta f$.