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Disturbance rejection problem solvability: From structural approach to reliability/availability analysis

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ABSTRACT

The aim of the paper is to study the impact of component failures on an important control problem for an automated system when operating: the disturbance rejection. The paper proposes a method based on a graph–theoretical approach to study the reliability and the availability of the controller capacity to reject some external disturbances. Only the system structure is considered known. The paper focuses on external and internal component failures and assumes that their probabilities are known. The satisfaction conditions of the disturbance rejection problem solvability for the structured linear systems are recalled. The first contribution of the paper is to express this property as a Boolean expression based on the functioning state of the involved component. A second contribution is to extend the definition of the reliability and the availability to cover the disturbance rejection ability and to assess the probability to conserve or to lose this property according to the components reliability and availability.

1. Introduction

In many applications, the disturbance rejection is one of the main purposes in controlled systems. Indeed, very often, some control objectives (stabilisation, tracking, etc.) must be satisfied on some system outputs whatever the external disturbances or faults are. For instance, the ability of the controlled system to reject the most important disturbances is of great interest in many automatic control problems as control law synthesis, fault tolerant control for stabilisation or desired trajectory tracking and so on. This implies that the system must satisfy some structural conditions since the design phase to provide an operational automated system to the customers.

In fact, systems design is a key issue in system engineering. It is the process of defining the architecture, components, modules, interfaces and data for a system to satisfy specified requirements given by the consumer during the functional phase. But, before this step, the dimensions of the equipment to design are usually unknown. Working with unknown dimensions can be cumbersome, but it is possible to work on the basis of generic models developed from the state equations.

To handle the disturbance rejection problem solvability, many works [2,18,22] deal with geometric and algebraic formalizations which necessitate the good and precise knowledge of system state space models. However, as previously mentioned state equations are not numerically dimensioned in early phases of the system life-cycle. Thus, the use of generic representations, called "structured models", defined by matrices which contain a fixed number of zero entries and other entries defined by free parameters determined on the basis of physical laws is very suitable. Many studies on structured systems are related to the graph–theoretical approach to analyze some system properties such as controllability, observability, fault diagnosticability, reconfigurability or the solvability of several classical control problems like disturbance rejection or input–output decoupling (see [4,10,16,17] and the references therein). The graph–theoretical approach provides simple and elegant analysis tools for these purposes. More precisely, the disturbance rejection problem has been tackled in [5,21] to establish the conditions of the disturbance rejection problem solvability. More recently, in [6,9] the authors classify the different existing sensors according to their importance for the solvability of such problem or finding where it is judicious to add sensors in order to recover the ability of the controlled system to reject the disturbances.

On the other hand, reliability engineers are concerned with the dependability properties of systems like Reliability, Availability, Maintainability and Safety (RAMS). In this paper, the reliability and availability analysis is considered. The reliability of a system

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measures how well it meets its design objectives during a mission time. It is fundamentally a probabilistic measure. It is expressed as a function of its components reliability. Thus, the question is to assess the reliability by defining a function called the "structure function" [11]. It allows quantifying the RAMS parameters accordingly through the system component probabilities. Availability is as important as reliability. It measures how well the system meets its objectives at each instant. Contrary to the reliability, availability considers restoration of components.

The edges in the system graphical representation are naturally linked to the internal and internal components of the system. External components are sensors and actuators, and internal can be a pipe, a damping, a pulley, a rolling, etc. according to the kind of the system (hydraulic, mechanical, etc.). Thus, the validity of system structural properties as the ability to reject disturbances can be impacted by component failures. Consequently, it seems clear that the two problems of conceiving automated reliable and available systems should be considered jointly and early in the system life-cycle in order to guarantee some confidence level on those properties by a probability assessment.

There exists many research works which deal with fault-accommodation and reconfiguration when actuators or sensors are prone to failures (see [20] for example). But, to our knowledge, there are only few works that deal with the interaction between automatic control systems and reliability/availability analysis in our way. In [3], the authors consider the sensor placement problem by combining a fault diagnosis observability study by signed directed graphs and reliability information on component failures probability. [13] also proposes to solve the sensor placement problem for diagnosis and introduces reliability and redundancy criterion to enhance the reliability of measurements. In [8,14], the authors consider the reliability of the observability and controllability properties of structured linear systems through a graph theoretical approach.

In this context, the contribution of the proposed paper is first to define the structural disturbance rejection conditions as a Boolean expression based on the component functioning states. The second contribution is to compute the reliability/availability of this property from the components reliability/availability through the corresponding structure function. The reliability computation can be achieved through a Markov chain.

Problem statement: Let us consider the following linear system:

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Eq(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, $q \in \mathbb{R}^d$ is the vector grouping the disturbances, $y \in \mathbb{R}^p$ is the output vector *i.e.* the output to be controlled, $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $E \in \mathbb{R}^{n \times d}$, $C \in \mathbb{R}^{p \times n}$ and $D \in \mathbb{R}^{p \times m}$ is the feedthrough matrix. The elements of all these matrices are either fixed to zero or assumed to be nonzero free parameters noted α_i. The set of these parameters is noted $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_h\}$. If all parameters α_i are numerically fixed, we obtain the so-called admissible realization of structured system Σ.

A structural property is generically satisfied if it is satisfied for almost all the realizations of the structured system Σ. Here, "for almost all the realizations" is to be understood as "for all parameter values ($\alpha \in \mathbb{R}^h$) except for those in some proper algebraic variety in the parameter space".

Solving the disturbance rejection problem consists of the generic existence of a feedback $u(t) = Fx(t)$ such that the closed loop system transfer matrix from the disturbance to the output is equal to zero, *i.e.* $G(s) = (C + DF) \cdot (sI - A - BF)^{-1} \cdot E = 0$. This problem is called the disturbance rejection problem by state feedback.

The objective of this paper is to compute the reliability and the availability of the structural property: disturbance rejection problem solvability. This represents the originality of the paper since reliability and availability are usually computed for systems and, to our knowledge, not for structural properties.

For this purpose, our study is carried into two steps. The first step of the proposed approach is to provide a Boolean expression based on the component functioning states which is equal to "true" when the disturbance rejection problem is solvable under the technical constraint that the generic rank of the transfer matrix $T(s) = C(sI - A)^{-1}B + D$ is equal to the number of outputs p. This constraint is quite natural in a control context. For all the reasons cited earlier in this section, we choose a graph–theoretical approach to address this problem.

The Boolean expression is a key point to compute the probability of the system ability to conserve the studied property satisfied given the reliability and availability of each involved component. For this purpose, Markov chains are used as a tool to compute the availability of the disturbance rejection problem solvability.

The paper is organized as follows. Section 2 is devoted to some definitions related to the graph–theoretical approach. The main results concerning the structural analysis of the disturbance rejection problem are exposed in Section 3. Reliability/availability analysis is developed in Section 4. Section 5 is dedicated to a case study before a brief conclusion.

2. Graphical representation of structured linear systems

2.1. Digraph definition for structured linear systems

Each linear system Σ can be associated to a directed graph (digraph). This digraph is denoted $\mathcal{G}(\Sigma)$ and represents the structure of the system *i.e.* its variables and their relationships. Digraph $\mathcal{G}(\Sigma)$ is constituted by a vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$. The vertices represent the variables of the system: states, control inputs, disturbances and outputs of Σ, whereas edges in $\mathcal{E}$ model the dynamic relations between these variables. More precisely, $\mathcal{V} = \mathbf{X} \cup \mathbf{U} \cup \mathbf{Y} \cup \mathbf{Q}$, where $\mathbf{X} = \{\mathbf{x_1}, \ldots, \mathbf{x_n}\}$ is the set of state vertices, $\mathbf{U} = \{\mathbf{u_1}, \ldots, \mathbf{u_m}\}$ is the set of control input vertices, $\mathbf{Y} = \{\mathbf{y_1}, \ldots, \mathbf{y_p}\}$ is the set of output vertices, and $\mathbf{Q} = \{\mathbf{q_1}, \ldots, \mathbf{q_d}\}$ is the set of input disturbance vertices. The edge set is $\mathcal{E} = \mathcal{E}_A \cup \mathcal{E}_B \cup \mathcal{E}_C \cup \mathcal{E}_D \cup \mathcal{E}_E$, with $\mathcal{E}_A = \{(\mathbf{x}_j, \mathbf{x}_i) | A(i,j) \neq 0\}$, $\mathcal{E}_B = \{(\mathbf{u}_j, \mathbf{x}_i) | B(i,j) \neq 0\}$, $\mathcal{E}_C = \{(\mathbf{x}_j, \mathbf{y_i}) | C(i,j) \neq 0\}$, $\mathcal{E}_D = \{(\mathbf{u}_j, \mathbf{y_i}) \ | D(i,j) \neq 0\}$ and $\mathcal{E}_E = \{(\mathbf{q}_j, \mathbf{x}_i) | E(i,j) \neq 0\}$.

Example 1. Consider the structured linear system defined by

$$A = \begin{pmatrix} 0 & 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_6 & 0 & 0 & 0 & \alpha_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_9 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} \alpha_{10} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \alpha_{12} & 0 \\ 0 & 0 & \alpha_{13} \\ 0 & 0 & \alpha_{14} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 \\ \alpha_{17} & 0 \\ 0 & 0 \\ \alpha_{18} & 0 \\ \alpha_{19} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \alpha_{20} \\ 0 & 0 \end{pmatrix},$$