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Identifiability of dynamic networks with part of the nodes noise-free

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Abstract: In dynamic network identification a major goal is to uniquely identify the topology and dynamic links between the measured node variables. It is common practice to assume that process noises affect every output in multivariable system identification, and every node in dynamic networks with a full rank noise process. For many practical situations this assumption might be overly strong. This leads to the question of how to handle situations where the process noise is not full rank, i.e. when the number of white noise processes driving the network is strictly smaller than the number of nodes. In this paper a first step towards answering this question is taken by addressing the case of a dynamic network where some nodes are noise-free, and others are disturbed with a (correlated) process noise. In this situation the predictor filters that generate the one-step-ahead prediction of the node signals are non-unique, and the appropriate identification criterion leads to a constrained optimization problem. It is assessed when it is possible to distinguish between models on the basis of this criterion, leading to new notions of network identifiability. It appears that a sufficient condition for network identifiability is that every node signal in the network is excited by an external excitation signal or a process noise signal that is uncorrelated with other node excitations.

Keywords: System identification, dynamic networks, network topology detection, reduced rank noise process, identifiability

1. INTRODUCTION

Interesting topics in the system identification field are the problems of topology detection and identification of dynamics in a dynamic network setting. In this setting the topology refers to the (boolean) interconnection structure in the network. Multiple authors have developed methods to perform identification in dynamic networks (Torres et al., 2015; Van den Hof et al., 2013; Dankers, 2014), and topology detection (Chiuso and Pillonetto, 2012; Yuan et al., 2011; Sanandaji et al., 2011; Materassi and Salapaka, 2012). In these publications the general setup is as follows: the dynamic network consists of dynamic links between (measured) node variables, (known) external excitation signals and unknown stochastic process noises. External excitation signals are not incorporated in all the methods. For all of the methods it is assumed that the process noise variables affecting different nodes are mutually uncorrelated, and that each node is affected by a process noise variable. This type of noise is referred to as a full-rank mutually uncorrelated process noise.

In many practical situations it can be unrealistic to assume full rank and uncorrelated process noise. An example of such a situation is ship modeling where multiple variables are affected by only one disturbance, the waves, see (Linder, 2014). In a situation like this, the spectral density matrix of the noise will typically be singular. Also in the classical closed-loop network, see Figure 1, with a noise-disturbed plant output and a controller output that is noise-free, the vector noise process will be singular. In (Weerts et al., 2015) the question has been addressed whether models (topology and dynamics) can be distinguished from each other for the case of networks with correlated full rank process noise. It has been shown in that paper that uniqueness of the detected topology is essentially an identifiability issue and that because of the noise correlations all node variables need to be treated simultaneously. Rather than decomposing the problem into several MISO problems, this requires the handling of a MIMO problem. The analysis in (Weerts et al., 2015) is however no longer valid in case the noise is of reduced rank.

In the current paper we treat the most simple situation of a reduced rank noise process, namely a network where part of the nodes are noise-free, and the remaining nodes are contaminated by a full rank process noise, while it is known up front which nodes are noise-free. The main question to address is: *In the considered situation of a dynamic network with (some) given noise-free nodes, under what conditions (on external excitation, network topology and model structures) can we distinguish between two network models on the basis of measured signals?*

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In the identification literature little attention is paid to rank reduced noise processes, even though the classical closed-loop system (Figure 1) has this property. Closed-loop identification methods typically work around the issue by either replacing the external excitation signal r by a stochastic noise process, as e.g. in the join-IO method (Caines and Chan, 1975), or by only focussing on identifying the plant model (and not the controller), as e.g. in the direct method. In econometrics dynamic factor models have been developed to deal with the situation of rank reduced noise (Deistler et al., 2015).

In the full rank noise case it is rather obvious that unique estimates can be obtained from network transfers that describe the mappings from external excitation signals to measurable node signals. In the singular noise case this is not obvious. Therefore we will set up a framework for identification in the singular situation, that we expect to be valid not only for the situation of noise-free nodes, but also for the more general situation of singular process noise.

In our approach we first formally define the network (section 2), after which the identification setup is formulated (section 3). The predictor is derived, and it is shown that the predictor filters are not unique due to the presence of noise-free nodes. An appropriate -constrained- identification criterion is introduced to deal with the noise-free signals and the resulting nonuniqueness of the predictor filters. In section 4, network identifiability is addressed and analyzed, after which two illustrative examples are provided (section 5). The Proofs of the results in this paper are collected in a report version of the paper (Weerts et al., 2016).

2. DYNAMIC NETWORK SETTING

In this paper a dynamic network consisting of L scalar *internal variables* or *nodes* w_j, $j = 1 \cdots L$ is considered. Only the first p nodes are affected by noise. This leads to a network defined by the equation

$$\begin{bmatrix} w_a(t) \\ w_b(t) \end{bmatrix} = \underbrace{\begin{bmatrix} G^0_{aa}(q) & G^0_{ab}(q) \\ G^0_{ba}(q) & G^0_{bb}(q) \end{bmatrix}}_{G^0(q)} \begin{bmatrix} w_a(t) \\ w_b(t) \end{bmatrix} \cdots \\ + \underbrace{\begin{bmatrix} R^0_a(q) \\ R^0_b(q) \end{bmatrix}}_{R^0(q)} r(t) + \underbrace{\begin{bmatrix} H^0_s(q) \\ 0 \end{bmatrix}}_{H^0(q)} e(t), \tag{1}$$

where:

- $G^0_{aa} \in \mathbb{R}^{p\times p}(z)$, $G^0_{ab} \in \mathbb{R}^{p\times(L-p)}(z)$, $G^0_{ba} \in \mathbb{R}^{(L-p)\times p}(z)$, $G^0_{bb} \in \mathbb{R}^{(L-p)\times(L-p)}(z)$ are proper rational transfer function matrices;
- nodes $w_a \in \mathbb{R}^p$, $w_b \in \mathbb{R}^{L-p}$;
- $\begin{bmatrix} H^0_s(q) \\ 0 \end{bmatrix} e(t)$ is the *process noise* affecting the nodes w_a, it is modeled as a realization of a stationary stochastic process with rational spectral density;
- $e(t) \in \mathbb{R}^p$, a stationary white noise process with diagonal covariance matrix $\Gamma > 0$;
- $R^0_a \in \mathbb{R}^{p\times K}(z)$, $R^0_b \in \mathbb{R}^{(L-p)\times K}(z)$, $K \in \mathbb{N}_0$;
- $r(t) \in \mathbb{R}^K$, it is the quasi-stationary *external excitation variable* that can directly or indirectly be manipulated by the user.

The diagonal of $G^0_{aa}(q)$ and $G^0_{bb}(q)$ is 0, i.e. nodes are not connected to themselves directly. There are no algebraic loops in the network, i.e. when individual elements of $G^0(q)$ are denoted by $G^0_{n_i n_j}(q)$ then for any sequence $n_1, \cdots, n_k$: $\lim_{z\to\infty} G^0_{n_1 n_2}(z) G^0_{n_2 n_3}(z) \cdots G^0_{n_k n_1}(z) = 0$. Nodes w_a are affected by noise, and nodes w_b are noise-free. $H^0_s(q)$ is square, monic, stable and stably invertible, $H^0_s \in \mathbb{R}^{p\times p}(z)$. Note that $\lim_{z\to\infty} H^0_s(z) = I$, such that the innovations process of the network is $e_0(t) := [I\ 0]^T e(t)$ where $e_0 \in \mathbb{R}^L$ which is in line with the definition of the innovations process in (Caines, 1987). For convenience we will denote the internal and external variables as

$$z(t) = \begin{bmatrix} w_a(t) \\ w_b(t) \\ r(t) \end{bmatrix}.$$

The topology of the network is defined as the set of indices that represent which interconnections in the network are nonzero.

Definition 1. Set $\mathcal{N}$ represents the boolean topology of (1), it is defined by

$$\mathcal{N} = \left\{ (n_i, n_j) \mid \exists\, z \text{ such that } G_{n_i,n_j}(z) \neq 0 \right\},$$

where n_i and n_j indicate the specific interconnection

$$n_i, n_j \in \{1, 2, \cdots, L\}.$$ □

For a dynamic network with noise-free nodes as in (1), the resulting identification problem then becomes to identify the topology and/or the network dynamics $\{G^0, H^0, R^0\}$ on the basis of measured node variables $\{w_j,\ j = 1, \cdots L\}$ and external variables $\{r_k,\ k = 1, \cdots K\}$. In this paper we will identify the topology through identification of the dynamic networks $\{G^0, H^0, R^0\}$.

We assume that it is known which nodes of a network are noise-free such that the network can be written in the form described above. In practice it is possible to estimate the covariance of noise in a network, hence it is possible to determine which nodes are noise-free. We can use that information to partition the nodes into the noisy w_a and noise-free w_b groups.

3. NETWORK PREDICTOR AND IDENTIFICATION CRITERION

In this section a prediction error identification setup is presented that is suited for dealing with the situation of noise-free nodes. This setup will be used to identify the network dynamics. All node signals in the network are treated symmetrically, i.e. no distinction is made between input and output node signals. First we define the one-step-ahead predictor as follows:

Definition 2. The one-step-ahead predictor is defined for $j = 1, \cdots, L$ as

$$\hat{w}_j(t|t-1) := \mathbb{E}\left\{ w_j(t) \mid w_j^{t-1},\ w_i^t\ \forall\, i \neq j,\ r^t \right\}$$

where $w_i^t := \{w_i(0), \cdots, w_i(t)\}$ and $r^t := \{r(0), \cdots, r(t)\}$. □

Although algebraic loops are not allowed, transfer functions without delay are allowed, which leads to the predictor expression above. Due to the non-square noise filter H^0 the expressions for the predictor will be different from the classical full rank case. This is shown in the following result.