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Nonlinear $\mathcal{H}_\infty$ Control for an Autonomous Underwater Vehicle in the Vertical Plane ⋆

Subhasish Mahapatra * **Bidyadhar Subudhi** * **Raja Rout** *
B. V. S. S. Krishna Kumar **

* *Centre for Industrial Electronics & Robotics, National Institute of Technology Rourkela, Odisha, India- 769 008 (e-mail: subhasish110@gmail.com, bidyadharnitrkl@gmail.com, and raja.rout86@gmail.com)*
** *Scientist-G, Control and Guidance Division, NSTL, Visakhapatnam India- 530 027 (e-mail: krishnakumar.bvss@nstl.drdo.in)*

Abstract: In this paper, a nonlinear path following control algorithm has been illustrated for Autonomous Underwater Vehicle (AUV) in the Vertical Plane. The nonlinear $\mathcal{H}_\infty$ state feedback control technique has been adopted to track the desired path in the dive plane. The design of the control algorithm is obtained by solving a Hamilton-Jacobi-Isaacs inequality with a Taylor's series approach for the control of AUV. The developed algorithm provides a significant performance in robustness and internal stability by attenuating the disturbance. The performance of the developed control scheme has been reported by carrying out the simulation using MATLAB/Simulink environment.

Keywords: Path following, AUV, Nonlinear Control, Nonlinear $\mathcal{H}_\infty$ Control, State Feedback

1. INTRODUCTION

In the current era, Autonomous Underwater Vehicles (AUVs) play a vital role in various oceanographic applications. The control of AUV in an oceanic environment possesses a huge challenge for researchers to develop robust control algorithms. The challenges such as exploration of uneven sea floor structure, diving control at shallow water condition, weak communicative link between the AUV and base station imposes uncertainties to the system. Hence, this leads to the notion of designing robust control techniques to tackle with these uncertainties. Thus, many linear controls (such as Subudhi et al. (2013) and You et al. (2010)) and nonlinear robust control strategies (Lapierre and Jouvencel (2008) and Naik and Singh (2007)) had been developed in the recent years to tackle with the above stated problems. As AUV involves a highly nonlinear system with 6-degree of freedom structure as shown in Fig. 1, accordingly it is intended to linearize the nonlinear model for the design of linear control algorithm. However, the linear controls were less effective towards addressing the challenges discussed earlier. As a consequence, a nonlinear $\mathcal{H}_\infty$ control algorithm is explored for the control of AUV in the Vertical Plane. The purpose of designing a $\mathcal{H}_\infty$ control is to achieve a robust and internally stable close loop system by eliminating the external perturbations.

The $\mathcal{H}_\infty$ control technique provides a significant contribution for the development of robust control algorithm. The solutions for different $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems were described elaborately in Doyle et al. (1989). The solutions obtained in Doyle et al. (1989) are specifically for linear systems but some important assumptions used in this are applicable for designing the nonlinear $\mathcal{H}_\infty$ control algorithm. In van der Schaft (1992), a nonlinear $\mathcal{H}_\infty$ control technique is elaborated using $\mathcal{L}_2$-gain analysis (a time domain approach of $\mathcal{H}_\infty$ norm). Nonlinear state feedback and measurement feedback $\mathcal{H}_\infty$ control algorithms were described in Isidori and Astolfi (1992), Isidori and Kang (1995). These control algorithms designed for disturbance attenuation problems with improved internal stability and robustness. Different applications of nonlinear $\mathcal{H}_\infty$ control were reported in Buppo et al. (1995), Sinha and Pechev (2004), Ferreira et al. (2008), Hioe et al. (2014). The purpose of using the nonlinear $\mathcal{H}_\infty$ control is to achieve a guaranteed performance in tracking the desired value by attaining strong internal stability and robustness. In Aliyu (2011), other approaches to find out the solutions for the HJI inequality have been discussed.

This paper illustrates the nonlinear $\mathcal{H}_\infty$ state feedback control law obtained by solving the Hamilton-Jacobi-Isaacs (HJI) inequality. A nonlinear 3-DOF structure of AUV has been considered for designing the control algorithm. The HJI inequality is formulated using the energy dissipative theory as explained in Isidori and Astolfi (1992). The control algorithm developed here attains a strong internal stability by attenuating the disturbance. The modelling of AUV is reported in Fossen (1994), Silvestre (2000). In this, the mathematical model of AUV is considered from Silvestre and Pascoal (2007). The usage of Taylor's series based robust control technique on AUV control is a unique feature, which has not been addressed in the AUV literature to date.

The introduction is followed by section 2 which highlights the nonlinear modeling of AUV in the Vertical Plane. The

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formulation of nonlinear $\mathcal{H}_\infty$ control algorithm for the control of AUV in section 3. Section 4 elaborates the numerical computation of the control algorithm developed for AUV. The analysis of simulation results and conclusions are explored in section 5 and 6 respectively.

2. AUV MODELING

This section illustrates the modeling of AUV in the Vertical Plane and formulation of an appropriate model for designing the nonlinear $\mathcal{H}_\infty$ state feedback control law for path following.

Consider the generalized structure of the nonlinear state space system that is described as follows

$$\dot{x}(t) = f(x) + g_1(x)d(t) + g_2(x)u(t) \tag{1a}$$

$$z(t) = k_1(x) + l_{11}(x)d(t) + l_{12}(x)u(t) \tag{1b}$$

$$y(t) = k_2(x) + l_{21}(x)d(t) \tag{1c}$$

where x, u, d, z and y are the state vector, the control input, exogenous inputs (which involves disturbances), penalty vector and measured variables respectively. The functions $f(x)$, $g_1(x)$, $g_2(x)$, $k_1(x)$, $k_2(x)$, $l_{11}(x)$, $l_{12}(x)$ and $l_{21}(x)$ are mapped smoothly of class C^∞, defined in a neighbourhood of the origin with following assumptions: $f(0) = 0, k_1(0) = 0$, and $k_2(0) = 0$.

The control of AUV is achieved by keeping a constant forward velocity. As a consequence, the surge motion equations is neglected from the dynamics. Let the state and disturbance vector be written as

$$x(t) = [w, q, z, \theta]^T = [x_1, x_2, x_3, x_4]^T,$$
$$d(t) = [d_1(t), d_2(t)]^T,$$

where $d_1(t)$ and $d_2(t)$ are the process noise vector and measurement noise associated with AUV dynamics. Thus, the dynamics of AUV is represented in terms of nonlinear structure (1a) with the functions such as

$$\begin{aligned}
f(x) &= [f_1(x), f_2(x), f_3(x), f_4(x)]^T,\\
f_1(x) &= C_w[(W - B)\cos x_4 + C_{Z_w} u x_1\\
&\quad + C_{Z_q} u x_2 + m u x_2],\\
f_2(x) &= C_q[Z_{CB} B \sin x_4 + C_{M_w} u x_1 + C_{M_q} u x_2],\\
f_3(x) &= -u \sin x_4 + x_1 \cos x_4,\\
f_4(x) &= x_2,
\end{aligned}$$

$$g_1(x) = \begin{bmatrix} C_w C_1 & 0 \\ C_q C_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, g_2(x) = \begin{bmatrix} C_w C_{Z_{\delta_s}} \\ C_q C_{M_{\delta_s}} \\ 0 \\ 0 \end{bmatrix},$$

$$u(t) = \delta_s, C_w = (m - C_{Z_{\dot{w}}})^{-1}, C_q = (m - C_{M_{\dot{q}}})^{-1},$$

where $C_{(*)}$ are the hydrodynamic coefficients as reported in Silvestre and Pascoal (2007) and C_1 and C_2 are the coefficients of disturbances in heave and pitch motion respectively. The penalty and measurement vectors are presented as

$$z(t) = K_1 x(t) + L_{12} u(t) \tag{2}$$

$$y(t) = K_2 x(t) + L_{21} w(t) \tag{3}$$

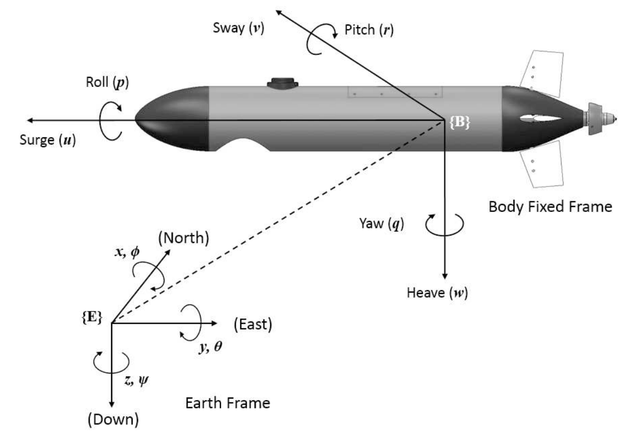

Fig. 1. Representation of AUV Body frame with respect to Earth frame

where

$$K_1 = \begin{bmatrix} W_1 & 0 & 0 & 0 \\ 0 & W_2 & 0 & 0 \\ 0 & 0 & W_3 & 0 \\ 0 & 0 & 0 & W_4 \\ 0 & 0 & 0 & 0 \end{bmatrix}, L_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ W_u \end{bmatrix}$$

$$K_2 = [0\ 0\ 1\ 0], L_{21} = [0\ C_3]$$

and C_3 is the coefficient of sensor noise during depth tracking. W_1, W_2, W_3, W_4 and W_u are the weights for the penalty variable $z(t)$ which is associated with x_1, x_2, x_3, x_4 and $u(t)$ respectively.

3. NONLINEAR STATE FEEDBACK $\mathcal{H}_\infty$ CONTROLLER

In this, the nonlinear $\mathcal{H}_\infty$ control algorithm is formulated by solving the HJI inequality. The L_2 - gain inequality is formulated here from the concept of dissipativity theory which suggests that the energy of the output signal (as penalty vector z) is smaller than the energy of input signal (as disturbance input d) and is written as

$$\int_0^{T_s} (\|z(t)\|^2 - \gamma^2 \|d(t)\|^2) dt \leq 0, \tag{4}$$

where γ varies between 0 and 1, which suggests that the system will be γ - dissipative, hence there is an existence of a nonnegative smooth storage function $S(x)$ which satisfies the following HJI inequality

$$\begin{aligned}
&S_x(x)^T [f(x) + g_1(x) d + g_2(x) u]\\
&\quad + [K_1 x + L_{12} u]^T [K_1 x + L_{12} u] - \gamma^2 d^T d \leq 0
\end{aligned} \tag{5}$$

where $S_x(x)^T = \partial S(x)/\partial x$.

This inequality will hold good for all d and T_s. Hence by choosing a proper positive semidefinite storage function, L_2- gain problem is solved which will satisfy equation (5). This equation is expressed as $\mathscr{H}[x, S_x(x), d, u]$ which is termed as Hamiltonian function. The objective of the controller is to achieve a significant steady state response with strong internal stability by attenuating the affect of the exogenous input w on the close loop system.