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On the Construction of a New Chaotic System

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Abstract: This paper puts forward the construction of a new chaotic system based on anti-control of chaos. The Bhalekar-Gejji (BG) chaotic system undergoes chaotification in order to generate chaos. Chaotification is achieved by destabilising the equilibria. The novelty of the paper is that the BG system is subjected to chaotification for the very first time to the best of the authors' knowledge. Numerical simulation is done in MATLAB environment. Simulated results reveal the successful achievement of the objective.

Keywords: Chaos Theory, Chaotic Behaviour, Bhalekar-Gejji Chaotic Systems, Anti-control of Chaos or Chaotification.

1. INTRODUCTION

Chaos control or anti-control refers to purposefully manipulating chaotic dynamical behaviours of a complex nonlinear system. As a new and young discipline, chaos control or anti-control has in fact played a major role in conventional scientific and technological advances today. Creating chaos in a system has become a key issue in many engineering applications where chaos is an important and useful phenomenon. Lorenz (1963) found the first chaotic attractor in a simple mathematical model of a weather system. In-depth researches have been carried out to generate new chaotic systems in the last four decades. Rossler (1976) conducted an important work that rekindled the interest in three-dimensional (3D) dissipative dynamical systems. This dynamical system is known as Rossler chaotic system. Then, many Lorenz-like or Lorenz based chaotic systems were proposed and investigated such as Lu and Chen (2002); Liu and Chen (2003); Liu et al. (2004); Celikovsky and Chen (2005); Qi et al. (2005, 2008); Zhou et al. (2008); Li et al. (2009); Dadras and Momeni (2009), etc.

Creating chaos as per our purpose is a nontrivial problem with interesting implications in both engineering applications and research community. Chen and Ueta (1999) introduced a new chaotic system using feedback control approach, is known as Chen system. Many such researches are available in the recent years like Wang et al. (2010); Pehlivan and Uyaroglu (2010); Jia et al. (2011); Wang et al. (2011); Wei and Yang (2011); Bhalekar and Gejji (2011); Yu and Wang (2012); Wang and Chen (2012); Pehlivan and Uyaroglu (2012); Sundarapandian and Pehlivan (2012); Li et al. (2013); Abooee et al. (2013), etc.

Given a non-chaotic system, which may be linear or nonlinear, the question whether one can generate chaos is very important. The process of generating chaos by means of designing a simple and implementable controller, e.g. a parameter tuner or a state feedback controller, is known as *anti-control of chaos* or *chaotification*. This problem is theoretically attractive and yet technically very challenging.

In finding of a new chaotic system, one can construct and determine the system parameters such that the system becomes chaotic following some basic ideas of chaotification, Wang (2003), as given below:

(1) It is dissipative.
(2) There exist some unstable equilibrium points, especially some saddle points.
(3) There are some cross-product terms, so there are dynamical influences between different variables.
(4) The system orbits are all bounded.

This motivates the present study on the problem of generating a new chaotic attractors. Under these guidelines, though not sufficient, a new chaotic system is generated by modifying the system given by Bhalekar and Gejji (2011).

A simple linear partial state feedback controller is designed to drive the stable BG system into chaotic mode. It leads to the discovery of a modified chaotic system, which is competitive with the BG system in the structure (it is a three-dimensional autonomous equation with only two quadratic terms), topologically not equivalent (there does not exist a homeomorphism that can map one system on the other). Further, the new chaotic system has even more

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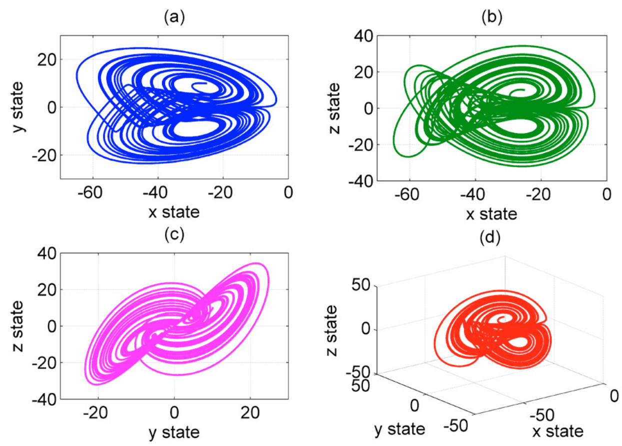

Fig. 1. Chaotic attractors of BG system.

complex dynamical behaviour than the original BG chaotic system.

After a brief introduction in Section 1, rest of the paper is organised as follows: Detailed description about the BG chaotic system is presented in Section 2. In the Section 3, formation of a new chaotic system using anti-control is given. Results and discussion are presented in Section 4. Finally, conclusions are drawn in Section 5.

In the next section an introduction to the BG chaotic system is reproduced for the sake of completeness.

2. DESCRIPTION OF BG CHAOTIC SYSTEM

BG chaotic system is reported by Bhalekar and Gejji (2011). BG system has different complex behaviour. The dynamics of 3D BG chaotic system is described as:

$$\begin{cases} \dot{x} = \omega x - y^2 \\ \dot{y} = \mu(z - y) \\ \dot{z} = \alpha y - \beta z + xy \end{cases} \quad (1)$$

where x, y, z are the states of the system (1). $\omega < 0$, and μ, α, β are the positive parameters of the BG system as given in Bhalekar (2012). The system (1) shows chaotic behaviour for $\omega = -2.667$, $\mu = 10$, $\alpha = 27.3$, $\beta = 1$. Phase plane behaviour of the system (1) is shown in Fig. 1. The equilibria of BG chaotic system are $E_1(0, 0, 0)$, $E_2(-26.3, 8.375, 8.375)$, and $E_3(-26.3, -8.375, -8.375)$. The Lyapunov exponents are calculated as $L_1 = 0.9694, L_2 = -0.008$, and $L_3 = -14.6452$. A positive lyapunov exponent reveals the presence of chaotic nature.

After a brief introduction of BG chaotic system in this section, formation of a new chaotic system is given in the next section.

3. FORMATION OF A NEW CHAOTIC SYSTEM

A new chaotic system is proposed based on BG chaotic system by Bhalekar and Gejji (2011) in the paper and shown to be more chaotic than many other well known chaotic systems. The BG chaotic system by Bhalekar and Gejji (2011) is considered here and its parameters are chosen in such a manner that the BG system behaves in a stable manner. Fig. 2 shows a phase portrait of the stable BG system with $\omega = -10$, $\mu = 55$, $\alpha = 37$, $\beta = 10$ in which an orbit started from an arbitrary initial point is attracted into a sink. Fig. 3 shows the dynamics of Lyapunov exponents and thus confirming the stable nature of the BG system with chosen parameters.

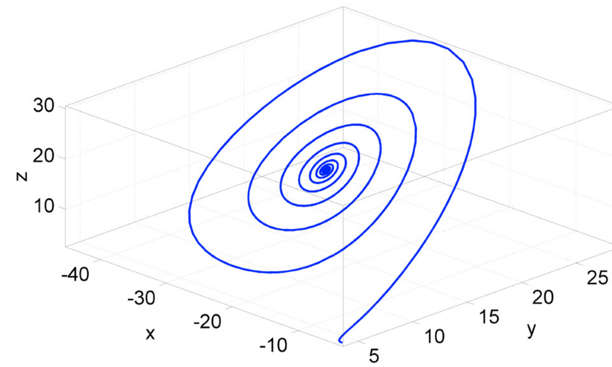

Fig. 2. Phase portrait of the stable BG system in $x-y-z$ space.

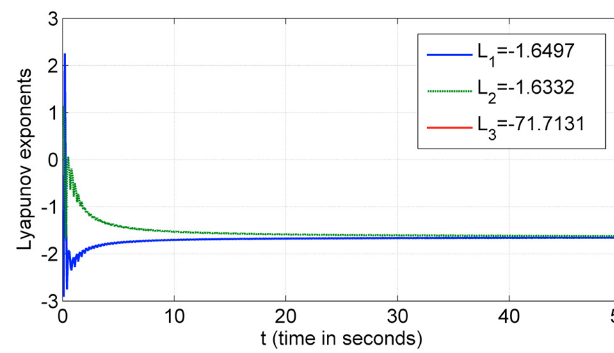

Fig. 3. Lyapunov exponents of the stable BG system.

A linear partial state feedback anti-control technique is used to create chaos. This method may not always yield a chaotic situation. Consider the controlled BG system (2) as:

$$\begin{cases} \dot{x} = \omega x - y^2 \\ \dot{y} = \mu(z - y) \\ \dot{z} = \alpha y - \beta z + xy + u \end{cases} \quad (2)$$

where ω, μ, α, β are constants, currently not in the range of chaos. Let us consider a linear state feedback controller as

$$u = k_1 x + k_2 y + k_3 z \quad (3)$$

where k_1, k_2, k_3 are unknown constant gains. Therefore, equation (2) becomes,

$$\begin{cases} \dot{x} = \omega x - y^2 \\ \dot{y} = \mu(z - y) \\ \dot{z} = k_1 x + (\alpha + k_2) y + (k_3 - \beta) z + xy \end{cases} \quad (4)$$

The Jacobian of (4) evaluated at (x^*, y^*, z^*) is given in (5).

$$J_{(x^*,y^*,z^*)} = \begin{pmatrix} \omega & -2y^* & 0 \\ 0 & -\mu & \mu \\ k_1 + y^* & \alpha + k_2 + x^* & k_3 - \beta \end{pmatrix} \quad (5)$$

Equilibrium points of the controlled BG system are given by solution of (6).