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Computing the reliability kernel of a time-variant system: Application to a corroded beam

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Abstract: Time-variant reliability analysis aims at assessing the probability of failure of a time-variant system within a given time horizon. We illustrate in this paper the computation of the reliability kernel which is the set of initial states for which the probability of failure remains under a threshold within the considered time horizon. This paper supposes that the time-variant system is discrete in time and space with given probabilities of transition between space states. We use a recursive relation for computing the cumulative probability of failure of the system, linking the probability of failure at time t with the probability of being at a given state x (for all possible states) at time $t-1$. Applying this relation, it is possible to compute the probability of failure at any starting point in the state space and hence to derive the reliability kernel. The computation of this kernel gives informations about the system which can be further helpful in reliable design. The approach is illustrated on an example of a steel beam under corrosion.

Keywords: Discrete systems, Dynamic systems, Initial states, Reliability analysis, Reliability kernel, System failures, Transition matrix, Reliable design

1. INTRODUCTION

The reliability analysis aims at computing the probability of failure of a system with respect to a defined failure measure and taking into account the uncertainties in the system's properties or context. Reliability theory comes from the field of industrial engineering (Banerjee, 1965; Barlow, 1984), but it is widely applied in different fields, from ecology (Maier et al., 2001) to industrial maintenance (Cazuguel and Cognard, 2006). This paper focuses on time-variant reliability which addresses the case of systems evolving in time.

Methods specifically developed for time-variant processes are mainly based on the time integration of the out-crossing rate, i.e. the rate at which the state may go through the limit state surface (Li and Der Kiureghian, 1997). Some algorithms follow a decomposition of the time-variant problem into a series of time-invariant ones (Hagen and Tvedt, 1991), leading to methods such as PHI2 (Andrieu-Renaud et al., 2004; Sudret, 2008).

We consider a time-variant system that is discrete in time and space, with given probability transitions between the space states (it is a Markovian process when these transition probabilities are constant in time) and we derive a recursive algorithm computing the failure probability within a time horizon from any starting point of the state space. This approach comes from the connection between time-variant reliability and viability theory (Aubin and Saint-Pierre, 2007) recently established in Rougé et al. (2014). It finally provides the reliability kernel, namely the set of all initial states for which probability of failure within the time horizon is below a given threshold. We propose to show an application of the computation of the reliability kernel in this paper, and its possible use in reliable design.

The paper is organized as follows. Section 2 introduces time-variant reliability notions such as the probability of failure, the reliability of the system and the reliability kernel. Then Section 3 shows the recursive algorithm built in order to get an approximation of this reliability kernel. After that, Section 4 proposes an application of a corroding steel beam in order to illustrate the approach. Finally, Section 5 concludes the paper.

2. RELIABILITY KERNEL

In this section, after introducing the concept of cumulated probability of failure, we present the notion of reliability kernel.

2.1 The stochastic system

Uncertainty is represented at each date by the random vector $\mathbf{W}(t)$. $\mathbf{W} = (\mathbf{W}(0), \mathbf{W}(1), ..., \mathbf{W}(T-1))$ is called a scenario and belongs to the set of all scenarios $\mathbb{S}$. The state of the system is a then a random vector $\mathbf{X}(t)$. In discrete time, we assume the following state transition between two consecutive dates t and $t+1$:

$$\mathbf{X}(t+1) = f(\mathbf{X}(t), \mathbf{W}(t)) \tag{1}$$

For the rest of this paper, we use the notation with realizations $x(t)$ of $\mathbf{X}(t)$ and realizations $w(t)$ of $\mathbf{W}(t)$:

$$f(x(t), w(t)) = x(t+1) \tag{2}$$

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for which $x(t)$ is a realization of $\mathbf{X}(t)$, $w(t)$ is a realization of $\mathbf{W}(t)$, and $x(t+1)$ is a realization of $\mathbf{X}(t+1)$.

The initial state is noted :

$$x_0 = x(0) \tag{3}$$

Contrary to Andrieu-Renaud et al. (2004) and Sudret et al. (2002), we made the assumption that the exact initial state x_0 of the system is known.

The system is discrete in space, the possible states being indexed by $i = \{1,..,N\}$. It is then possible to derive from Eq.1 the probability transitions between discrete states, and the probability that the system is at state x_j at time $t+1$ from probabilities of being at any state x_i $(i = \{1,..,N\})$ at time t:

$$\mathbb{P}(x(t+1) = x_j) = \sum_{i=\{1,..,N\}} \theta_{i,j} \mathbb{P}(x(t) = x_i). \tag{4}$$

The matrix $\theta_{i,j}$ is obtained from Eq.1 by:

$$\theta_{i,j} = \mathbb{P}(f(x_i, w(t)) = x_j). \tag{5}$$

When the matrix $\theta_{i,j}$ is constant in time, the system follows a Markovian process.

At each step of time t, the limit state surface $g(t, \mathbf{X}(t)) = 0$ separates the failure domain $F(t)$ and the survival domain $S(t)$.

2.2 Probability of failure and reliability

The definition of the cumulative probability of failure $P_{f,c}$ between two dates t_0 and t_1 $(0 \leq t_0 \leq t_1 \leq T)$ and knowing the state x_0 at time t_0 is:

$$P_{f,c}(t_0, t_1, x_0) = \mathbb{P}(\exists t \in \{t_0, t_0+1, ..., t_1\}, \mathbf{X}(t) \in F(t) \,|\, \mathbf{X}(t_0) = x_0) \tag{6}$$

This is the probability that at least one failure occurs during the planning period, knowing the initial state of the system. A distinction is made here between this cumulative probability of failure and the concept of instantaneous probability of failure. The former is calculated all over an interval of time, whereas the latter is computed by freezing time in the limit state function (Andrieu-Renaud et al., 2004).

The complement of the cumulative probability of failure is called reliability or probability of safety P_s:

$$P_s(t_0, t_1, x_0) = \mathbb{P}(\forall t \in \{t_0, t_0+1, ..., t_1\}, \mathbf{X}(t) \in S(t) \,|\, \mathbf{X}(t_0) = x_0) \tag{7}$$

This is the probability that the system belongs to the survival set during the whole planning period. The cumulated failure probability and the safety probability are linked by :

$$P_s(t_0, t_1, x_0) = 1 - P_{f,c}(t_0, t_1, x_0) \tag{8}$$

2.3 Reliability kernel

The cumulative probability of failure $P_{f,c}(0, T, x_0)$ is the probability for a trajectory $(x_0, x(1), ..., x(T))$ to leave the survival set over the planning period $[0, T]$. The set of all reliable initial states at a significance level α is called the reliability kernel :

$$Rel(\alpha, T) = \{x_0 \in S(0) \,|\, P_{f,c}(0, T, x_0) \leq \alpha\} \tag{9}$$

It is important to note that $P_{f,c}(0, T, x_0)$ depends on the time horizon and the initial state x_0. Moreover, we rewrite the definition of the reliability kernel by using the safety probability P_s defined in Eq.7 :

$$Rel(\alpha, T) = \{x_0 \in S(0) \,|\, P_s(0, T, x_0) \geq 1 - \alpha\} \tag{10}$$

We aim at computing the set $Rel(\alpha, T)$ by a recursive algorithm. We focus on the computation of the safety probability for the case where $x_0 \in S(0)$. The concept of reliability kernel was first proposed in Rougé et al. (2014). Note that contrary to this paper, we don't consider a controlled system here.

3. APPROXIMATION OF THE RELIABILITY KERNEL

The computation is made all over a grid of the possible initial states.

3.1 Recursion on the safety probability

We are looking for a backward recursive relation between $P_s(t, T, x_i)$ and the safety probability $P_s(t+1, T, x_k)$ one time step ahead, denoting x_i a realization of $\mathbf{X}(t)$ and x_k a realization of $\mathbf{X}(t+1)$. According to Eq.7, $\forall t \in \{0, 1, ..., T-1\}$, $\forall x_i \in S(t)$:

$$P_s(t, T, x_i) = \mathbb{P}(\forall \tau \in \{t, t+1, ..., T\}, X(\tau) \in S(\tau) \,|\, X(t) = x_i) \tag{11}$$

Starting at state x_i at t and knowing that $x_i \in S(t)$, we write $P_s(t, T, x_i)$ as the probability to go from x_i to another state x_k which is in $S(t+1)$ and having a safety probability $P_s(t+1, T, x_k)$:

$$P_s(t, T, x_i) = \sum_{x_k \in S(t+1)} \theta_{i,k} P_s(t+1, T, x_k) \tag{12}$$

According to Eq.8, we now recursively express $P_{f,c}(t, T, x_i)$ regarding $P_{f,c}(t+1, T, x_k)$:

$$P_{f,c}(t, T, x_i) = 1 - \sum_{x_k \in S(t+1)} \theta_{i,k} \left(1 - P_{f,c}(t+1, T, x_k)\right) \tag{13}$$

Once we have this result, we recursively compute the approximation of the cumulated probability of failure all over the states. The reliability kernel is derived directly because state x_k belongs to the reliability kernel if the cumulated probability of failure $P_{f,c}(t, T, x_k)$ is below the threshold α. Note that Eq.12 leads to the recursive equation used in stochastic viability theory (Rougé et al., 2014).

4. APPLICATION

We use an adapted and simplified model from Sudret et al. (2002) to present our results.

4.1 Beam under loading

Let us consider a steel bending beam with a rectangular cross-section. Its length is $L = 5$ m and we note b_0 for the initial breadth and h_0 for the initial height of the beam. It is submitted to a dead load (noted $\rho_{st} = 7850$ kg/m the steel mass density, this load is $p = \rho_{st} b_0 h_0$ (N/m)) as well as a pinpoint load F applied onto its middle point (see Fig.1).