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Efficient Computation Method for Strong Stability Area of Neutral Equations

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Abstract: A reliable method for the computation of the strong stable area of neutral delay differential equations is presented. A special neutral system with commensurate delays is analysed. Phase parameters are used to determine all possible parameter points where a critical characteristic root may occur for a given time delay ratio. An extra condition is formulated to define the boundary curves of the robust stable area. The corresponding co-dimension 3 parameter problem in the 4 dimensional parameter space is solved efficiently by the Multi-Dimensional Bisection Method. Finally the so-called instability gradients are used to classify the distinct domains of the chart. It is shown, that the computational time of the proposed method to obtain the strong stable area is comparable to the CPU time needed for the computation of the stability chart for a given fixed delay ratio.

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1. INTRODUCTION

The determination of the stability of dynamical systems described by neutral equations has a long history [Grammatikopoulos et al. (1986); Sficas and Stavroulakis (1987); Graef et al. (1991); Sipahi and Olgac (2006); Olgac et al. (2008); Michiels et al. (2009); Sipahi et al. (2010); Henrion and Vyhldal (2012); Cesari et al. (2014)]. Such systems can occur in different fields of engineering problems [Bellen et al. (1999); Murray et al. (1998); Niculescu and Brogliato (1999)], for which the determination of stability conditions is of high importance. For dynamical systems where only the states are delayed, the stability properties can be well defined by the approximated characteristic roots based on methods shown in [Stepan (1989); Insperger and Stepan (2011); Hill (1886); Nayfeh and Mook (1979); Khasawneh et al. (2010)]. However, if the derivative terms are delayed, too, the characteristic roots may be sensitive to arbitrarily small perturbations of the time delays [Melvin (1974); Logemann and Townley (1996); Hale and Lunel (2001a); Michiels et al. (2002, 2009)]. The parameter ranges for which the neutral system is stable for any time delay perturbation are so-called strong stable, also referred to as robust stable or delay independent stable. The computation of these parameters is complicated due to the infinite sensitivity of the characteristic roots, thus special methods have to be applied. For the stability computation of systems with free delay parameters the Cluster Treatment of Characteristic Roots (CTCR) method is an appropriate choice [Olgac and Sipahi (2004); Sipahi (2005); Sipahi and Olgac (2006); Olgac et al. (2006, 2008); Sipahi et al. (2010)] or algorithms presented in [Jarlebring (2007); Péics and Karsai (2002)] can be applied. In [Michiels et al. (2009)] the delay dependency structure is also considered. Neutral functional differential equations are analysed in [Rabah et al. (2012)], while in [Bellen and Guglielmi (2009)] state-dependent delays are also described.

In this paper we consider a special neutral system with commensurate delays, for which all the delays are integer multiples of certain base delays. This model can describe mechanical systems with acceleration feedback. In paper [Insperger et al. (2010)], for instance, the tilt angle coordinate of a segway model is measured by an accelerometer. The angular velocity is approximated by a finite difference between some delayed values of the acceleration. This way, higher order finite difference schemes in the measured acceleration values can lead to neutral equations with delays which are strictly integer multiples of the sampling time. Another example is a digital acceleration feedback with distributed delay in which the integral along the range of the time delay is realized by a finite sum.

In this paper we focus on the stability of the corresponding difference equation of the neutral governing equation. An efficient method for the computation of the Strong Stable Area of the parameter space is presented.

2. NEUTRAL DELAYED SYSTEM

Based on the description above, a neutral system is considered to include multiple commensurate delays [Michiels et al. (2009)] (integer multipliers of a parameter).

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(x(t)+a\sum_{k=1}^{N_a}\hat{a}_k x(t-k\tau_a)+b\sum_{l=1}^{N_b}\hat{b}_k x(t-l\tau_b)\right)+ \quad (1)$$
$$x(t)+c\sum_{k=1}^{N_c}\hat{c}_k x(t-k\tau_a)+d\sum_{l=1}^{N_d}\hat{d}_k x(t-l\tau_b)=0.$$

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To ensure the stability of the neutral equation, the necessary (but not sufficient) condition must be fulfilled: the equation formed by the derivative terms of Eq. (1) (see its first line) have to be stable [Cesari et al. (1976); Hale and Lunel (2001b)]. In the present paper we will focus on the stability of this difference equation only:

$$x(t) + a\sum_{k=1}^{N_a} \hat{a}_k x(t - k\tau_a) + b\sum_{l=1}^{N_b} \hat{b}_k x(t - l\tau_b) = 0. \qquad (2)$$

In our test case, treated in this study, the stability computations were carried out for free control parameters a and b (parameters of the stability charts) and fixed coefficients $\hat{a} = [2, -1]$ and $\hat{b} = [1]$, which refer to a selected differential scheme of the control. Note, that in a general case $\hat{a}$ and $\hat{b}$ can be arbitrary vectors.

3. STABLITY LIMITS FOR FIXED DELAYS

In case, the exact values of the delays are known, the characteristic function $D(\lambda)$ of Eq. (2) can be found by substituting the trial solution $x(t) = e^{\lambda t}$ according to:

$$D(\lambda) := 1 + a\sum_{k=1}^{N_a} \hat{a}_k e^{-k\lambda\tau_a} + b\sum_{l=1}^{N_b} \hat{b}_k e^{-l\lambda\tau_b}. \qquad (3)$$

The stability boundaries are determined for the critical values of the roots $\lambda = i\omega_c$:

$$D(\omega_c) = 1 + a\sum_{k=1}^{N_a} \hat{a}_k e^{-ik\omega_c\tau_a} + b\sum_{l=1}^{N_b} \hat{b}_k e^{-il\omega_c\tau_b}. \qquad (4)$$

A co-dimension 2 problem is defined by the real and imaginary part of the characteristic equation:

$$\mathrm{Re}\left(D(a, b, \omega_c)\right) = 0 \qquad (5)$$

$$\mathrm{Im}\left(D(a, b, \omega_c)\right) = 0, \qquad (6)$$

in the three dimensional parameter space (a, b, ω_c) ($\hat{a}_k$ and $\hat{b}_l$ are considered to be constant). The so-called Multi-Dimensional Bisection Method (MDBM) [Bachrathy and Stepan (2012); Bachrathy (2012)] is a numerical computation algorithm designed to find the submanifolds of the roots of a system of non-linear equations. It can even be applied for higher parameter dimensions and co-dimensions. The roots of (5) and (6) are determined by the MDBM and are presented in Fig. 1 for different time delay ratios (τ_a/τ_b).

If the resulting numerator and denominator form of the delay ratio τ_a/τ_b contains only 'small' integer numbers (as is the case for the systems presented in the top row in Fig. 1), then the computation is not sensitive for the resolution of ω_c and a smaller range of ω_c is sufficient for the analysis. Consequently, in these cases the computation time is relatively small (for the final resolution of a and b 97x97 it is 2-5 seconds using Matlab 2014b, Intel Core i7-4710HQ CPU 2.70 GHz, 16 GB memory). Meanwhile, for $\tau_a/\tau_b = 10.01$ the CPU time is two orders of magnitude higher (134 s) and the resultant chart is still fragmented.

The results presented in Fig 1 show the (infinite) sensitivity to the delay ratio, which means that a small perturbation or the slightest uncertainty can change the stability chart completely. In case τ_a/τ_b is irrational, infinitely many bifurcation lines would occur and the range of ω_c during the computation would have to be infinity large.

4. INDEPENDENT PHASE PARAMETERS

If the periodicity of the exponential terms is considered in Eq. (4), then the phase parameters $\Phi_1 = \mathrm{mod}(\omega_c\tau_a, 2\pi)$ and $\Phi_2 = \mathrm{mod}(\omega_c\tau_b, 2\pi)$ can be introduced (similarly to [Michiels et al. (2009)]). During the numerical analysis it is sufficient to analyse the range $[0, 2\pi]$ for both parameters. If further symmetry properties are considered it might be enough to analyse the range $[0, \pi]$ for one of the variables.

This idea is applied in the CTCR method [Sipahi et al. (2010)], where the time delays τ_a and τ_b are the independent variables. In this study the idea is slightly different: even though Φ_1 and Φ_2 are connected through ω_c, they can be treated as independent variables due to the infinite sensitivity of the characteristic roots. In other words, if we consider an infinitesimal perturbation in τ_2 and take into account the infinite range of ω_c $[0, \infty]$, then Φ_2 can have any arbitrary value on the interval $[0, 2\pi]$, hence it is independent from Φ_1.

The characteristic function (4) can be rewritten with the help of the phase parameters:

$$D(a, b, \Phi_1, \Phi_2) = 1 + a\sum_{k=1}^{N_a} \hat{a}_k e^{-ik\Phi_1} + b\sum_{l=1}^{N_b} \hat{b}_k e^{-il\Phi_2}. \qquad (7)$$

For the 4 dimensional parameter space and co-dimension 2 problem, the resultant roots form a surface, which contains all possible parameter sets for (a, b) where critical characteristic roots ($\lambda = \mathrm{i}\omega$) can occur for a given delay scenario. This surface can be computed by means of MDBM. It can be presented in an impressive chart (see Fig.4), however, even with MDBM it requires large computation effort and only its boundary in the (a,b) parameter plane contains relevant information.

5. EXTRA CONDITIONS

The boundary of the strong stable area is defined by the envelope of this surface. As presented in [Bachrathy (2015)], in the vicinity of these parameter points, the real part of the roots λ of the characteristic equation of (3) do not change as a function of the perturbation parameter Φ_2, which is now considered as the perturbed parameter. This condition can be described as follows:

$$\mathrm{Re}\left(\frac{\partial\lambda}{\partial\Phi_2}\right) = 0 \qquad (8)$$

in which we focus on the critical value $\lambda = i\omega_c$, which is linearly depends on Φ_1. The left hand side of Eq. (8) can be determined by the implicit derivation of (7) [Stepan (1989)], and after the rearrangement of the terms one ends up with:

$$\mathrm{Re}\left(\frac{\partial\Phi_1}{\partial\Phi_2}\right) = \mathrm{Re}\left(-\frac{\frac{\partial D}{\partial\Phi_2}}{\frac{\partial D}{\partial\Phi_1}}\right) = 0. \qquad (9)$$

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