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Strong Structural Non-Minimum Phase Systems

Andreas Daasch *,** **Ferdinand Svaricek** ** **Matthias Schultalbers** *

* *IAV GmbH, 38518 Gifhorn, Germany (e-mail: {andreas.daasch, matthias.schultalbers}@iav.de).*
** *Institute of Control Engineering, Universität der Bundeswehr München, 85577 Neubiberg, Germany (e-mail: ferdinand.svaricek@unibw.de)*

Abstract: Structural analysis can reveal properties of the investigated systems without knowing the exact parameters of the system, for example masses, geometries or resistor values. Properties that can be determined are for instance controllability and observability. Recently, there have been investigations about structural stability and structural non-minimum phase behavior. In general by considering the structure of a system a precise answer if the mentioned properties hold numerically can not be given. This led to the introduction of strong structural properties, that hold for all admissible numerically realization of the analyzed system. In this paper the strong structural non-minimum phase property is investigated. The representation of a dynamical system as a graph is proposed for the structural analysis. A method to calculate the invariant zeros polynomial from the graph-theoretic representation of a square MIMO systems is introduced. From that method the strong structural non-minimum phase property is derived. An example for the application of the method is given.

Keywords: Structural properties, Control system analysis, Graph theory, Non-minimum phase systems, Zero dynamics

1. INTRODUCTION

The structure of a system is described by the mutual dependency of the state variables x_i the inputs u_i and the outputs y_i. That means, it is a "yes or no" criterion if e. g. a state variable x_i depends on another state variable x_k or an input u_j. A formal definition for the structure of a system and structurally equivalent systems is given next.

Definition 1. For a set of linear systems $\dot{x}(t) = A_i x(t) + B_i u(t)$, $y(t) = C_i x(t)$ with the same number of state variables, inputs and outputs, their common *structure* can be defined by matrices, A^*,B^* and C^*. An element in these matrices is zero, denoted by 0, if the element at the same position is identical zero respectively for all A_i,B_i and C_i. Otherwise this element is nonzero, denoted by $*$, if the element at the same position is nonzero and generally independent respectively for all A_i,B_i and C_i. Systems have an identical structure, if their states, inputs and outputs have identical dependencies of each other. Such systems are called *structurally equivalent.*

There have been many publications investigating the structure of a system. One of the first were Lin (1974) considering structural controllability and Iri et al. (1972) considering structural "solvability". The results for structural controllability and observability, finite and infinite zeros and poles, for linear systems where summarized in the book by Reinschke (1988) and the survey by Dion et al. (2003). In the book by Murota (2009) the topic of structural properties is discussed in a more mathematical sense. Structural properties like differential rank, infinite zeros and invertibility for nonlinear systems were described in the book by Wey (2002). Stability in structural systems was investigated by Belabbas (2013).

Obviously, some information about a system is lost if only its structure is considered. Therefore not always a precise answer whether the mentioned system properties hold numerically can be given. The advantage of the structural analysis is that, if a property holds structurally, it holds for almost all systems of the same structure. The question of the existence of structural properties which hold for all numerical realizations of a system arose. These properties were called strong structural properties (Mayeda and Yamada (1979)).

Definition 2. A *strong structural property* of a system is a property of a class of systems that are structurally equivalent. For this class the property under investigation holds numerically for all admissible numerical realizations.

Some former results about strong structural controllability were reinvestigated by Jarczyk et al. (2011) and shown to be wrong. This lead to a new graph-theoretic characterization of strong structural controllability. Recently Reißig et al. (2014) have extended the results about strong structural controllability of Mayeda and Yamada (1979) to the time-variant case. Chapman and Mesbahi (2013) investigated this property also for networked systems. The contribution of this work is to establish a strong version

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of the structural non-minimum phase property, which was established by the authors Daasch et al. (2016).

This paper is organized as follows. In the next section we will give a short introduction to the representation of dynamical systems as a graph and we will show how to determine the invariant zeros polynomial of the Rosenbrock's system matrix using graph-theoretical methods. With that the strong structural non-minimum phase property is stated in Section 3. An illustrative example of a strong structural non-minimum phase system is given in Section 4. In Section 5 we draw a conclusion and give an outlook to further research.

2. PRELIMINARIES

The linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \ , \end{aligned} \tag{1}$$

is considered where the system state variables are denoted by $x(t) \in \mathbb{R}^n$, the input vector by $u(t) \in \mathbb{R}^m$ and the output vector by $y(t) \in \mathbb{R}^p$.

A *graph* $\mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a set of *vertices* $v \in \mathcal{V}$ and a set of *edges* $e \in \mathcal{E}$. The graph exposes a structure by connecting two vertices, v_i and v_j, with an edge $e_{i,j}$. In a *directed* graph, edges establish a connection between two vertices and assign its direction. This can be extended to a *directed weighted* graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$ which additionally contains a set of *weights* $w \in \mathcal{W}$. In the weighted graph a value $w_{i,j}$ is assigned to every edge $e_{i,j}$. Graphically, vertices are represented by circles ◯ and edges by arrows $\rightarrow$ connecting the cycles and revealing the direction of connection. If the graph is weighted the labels of the edges reflect the value of their weights.

The dynamical system (1) can be represented as a graph using the following rules: The directed (weighted) graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ ($\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$) of the system (1) consists of m input vertices $u_1, \ldots, u_m$, n states vertices $x_1, \ldots, x_n$ and p output vertices $y_1, \ldots, y_p$. The vertices are connected by directed (and weighted) edges, generated by following rules:

(1) There exists a directed edge from input vertex u_j to state vertex x_i if in B the element $b_{i,j}$ in the i-th row and j-th column is nonzero. (Then the weight $w_{i,j}$ of the edge is given by $b_{i,j}$.)
(2) There exists a directed edge from state vertex x_j to state vertex x_i if in A the element $a_{i,j}$ in the i-th row and j-th column is nonzero. (Then the weight $w_{i,j}$ of the edge is given by $a_{i,j}$.)
(3) There exists a directed edge from state vertex x_j to output vertex y_i if in C the element $c_{i,j}$ in the i-th row and j-th column is nonzero. (Then the weight $w_{i,j}$ of the edge is given by $c_{i,j}$.)

A simple example of a graph-theoretic representation of a system is given in Fig. 1.

The unweighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ of system (1) reveals the structure of the system, since the edges in the graph coincide with the nonzero entries in the matrices A, B and C. From this point of view we can conclude the following.

Corollary 1. Systems that are structurally equivalent have the same unweighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$.

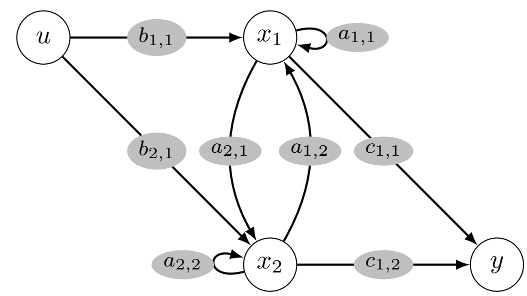

Fig. 1. Example of a weighted directed graph associated with a general SISO system of order 2.

Hence structural properties can by analyzed using the unweighted graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ and a structural property is valid for all systems that have the same unweighted graph.

Further, it exists a method to calculate the invariant zeros polynomial of a system from the graph-theoretic representation of it. We consider the system (1) with the restriction that the number of outputs equals the number of inputs, i. e. $m = p$. Such a system is called *square*.

The system (1) can be represented in the frequency domain by

$$\begin{bmatrix} X_0 \\ Y(s) \end{bmatrix} = \underbrace{\begin{bmatrix} sI - A & \text{-}B \\ C & 0 \end{bmatrix}}_{P(s)} \begin{bmatrix} X(s) \\ U(s) \end{bmatrix} \tag{2}$$

where $P(s)$ is called the Rosenbrock's system matrix. The invariant zeros s_{0_i} (MacFarlane and Karcanias (1976)) of a *non-degenerated* square system (1) (i. e. where rank $P(s) = n+m$ holds for allmost all s) are the roots of the invariant zeros polynomial

$$\begin{aligned} \det P(s) &= p_m s^{n-m} + p_{m+1} s^{n-m-1} + \ldots + p_{n-1} s + p_n \\ &= \sum_{k=m}^{n} p_k s^{n-k} \ . \end{aligned} \tag{3}$$

In order to calculate (3) form the graph-theoretic representation of a system, we need to define some structures in the graph.

Definition 3. A (directed) *path* in the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a sequence of edges $\{e_{i,j}, e_{j,k}, \ldots\}$ connecting the vertices $\{v_i, v_j, v_k, \ldots\}$ in forward direction, wherein every vertex is visited only once. If the first vertex and the last vertex of a path are identical the sequence is called a *cycle*. The *length* of a cycle is the number of vertices connected by it. A *cycle family* is the set of disjoint cycles in the graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, i. e. cycles that do not share vertices. The *width* of a cycle family is the number of *state* vertices it touches. A cycle family is *unique* if there exists no other cycle family of same width.

The coefficients of the polynomial (3) can be constructed by the minors of (2). To achieve this, the idea is to modify the open loop graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ in extension to Definition 31.1 by Reinschke (1988) as follows.

Definition 4. The *feedback graph* $\mathcal{G}_f(\mathcal{V}, \mathcal{E}, \mathcal{W})$ is constructed by inserting *feedback edges* in the graph $\mathcal{G}$ of system (1). The feedback edges connect the inputs u_i to the outputs y_i by the feedback law

$$u = \text{-}Iy \ , \tag{4}$$