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Robustness Properties of Distributed Configurations in Multi-Agent Systems ⋆

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Abstract: This paper considers a distributed LQR design framework for a multi-agent network consisting of identical dynamically decoupled agents. A systematic method is presented for computing the performance loss of various distributed control configurations relative to the performance of the optimal centralized controller. Necessary and sufficient conditions have been derived for which a distributed control configuration pattern arising from the optimal centralizing solution does not entail loss of performance if the initial vector lies in a certain subspace of state-space which is identified. It was shown that these conditions are always satisfied for systems with communication/control networks corresponding to complete graphs with a single link removed. A procedure is extended for analyzing the performance loss of an arbitrary distributed configuration which is illustrated by an exhaustive analysis of a network consisting of agents described by second-order integrator dynamics. Presented results are useful for quantifying performance loss due to decentralization and for designing optimal or near-optimal distributed control schemes.

Keywords: Distributed control, Linear quadratic regulator, Multi-agent systems, Robustness, Performance cost.

1. INTRODUCTION

Distributed coordination of multi-agent systems has received considerable attention in recent years. This problem has broad applications in many fields, e.g. unmanned aerial vehicles, distributed sensor networks and congestion control in communication networks (see Ren and Beard (2010), Jadbabaie et al. (2003), Cortes et al. (2004) and Paganini et al. (2001)). Typically in this type of systems, agents interact with each other in a distributed manner through local information exchange in order to achieve a common objective.

Literature tends to favour distributed control of multi-agent systems due to its advantages over centralized and decentralized control methods which become infeasible or unpractical as the number of agents and distance between them increases. An overview of these three control methods in multi-agent systems is given in Massioni and Verhaegen (2009). Often the problem of controlling the multi-agent system is combined with the graph theory where the information exchange is represented in terms of a graph, see e.g. Langbort et al. (2004) and Lin et al. (2007). Further, Fax and Murray (2004) proposed the graph theory based method for analysis of a formation of interacting and cooperating identical agents. The authors analyzed necessary and sufficient stability conditions for a given undirected communication topology. This framework was extended in Popov and Werner (2009) to the robust formation control method which is applicable to an arbitrary communication topology.

In general, optimal control of agents formations is a topic of considerable interest to the control community. It can be carried out to minimize the relative formation errors (Zhang and Hu (2007)), energy expenditure (Bhatt et al. (2009)), etc. Linear Quadratic Regulator (LQR) has been widely used in vast variety of scenarios due to its guaranteed robustness properties. For example, in Rogge et al. (2010) LQR-based method is proposed as a solution to the consensus problem over a ring network. Similarly, in Cao and Ren (2010) LQR theory has been successfully applied to control of multi-agent systems with single-integrator dynamics. Furthermore, the problem of controlling a formation of interacting and cooperating systems by employing a distributed LQR design was considered in Huang et al. (2010). Distributed LQR framework was also used to control a collection of identical dynamically coupled systems in Deshpande et al. (2011) and Borrelli and Keviczky (2008). Proposed control laws guarantee a certain level of performance in terms of LQR cost at network level, but they use different LQR cost functions. In Deshpande et al. (2011) the solution is depended on the total number of agents, while in Borrelli and Keviczky (2008) the solution is derived as a function of the maximum vertex degree.

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In the present work we use the distributed LQR design strategy for dynamically decoupled multi-agent systems introduced in Borrelli and Keviczky (2008). By solving a simple local LQR problem whose size is limited by the maximum vertex degree, a stabilizing distributed controller can be found. The aim of this paper is to compare the family of the distributed suboptimal controllers that has been introduced in Borrelli and Keviczky (2008) with the optimal centralized controller. It is shown that for any distributed control configuration which differs from a complete graph by a single link, there is no performance loss if the initial vector lies in a certain subspace of state-space. Additionally, the near-optimal schemes can be identified. A procedure is extended for analyzing the performance loss of an arbitrary distributed configuration which is illustrated by an example in which individual agents are described by second-order integrator dynamics. The results presented allow the application of the method described in Borrelli and Keviczky (2008) to decentralized control schemes optimized with respect to the structure.

The remainder of this paper is organized as follows. Section 2 defines the notations used in paper, which is followed by a brief summary of relevant results from graph theory. In Section 3 two different LQR control designs, centralized and distributed, are proposed. The main results of the paper are described in Section 4. Distributed control configurations that do not entail loss of performance relative to the LQR optimal centralized controller are identified. Also, the method for finding near-optimal distributed configurations for an arbitrary network is presented. These results are illustrated in Section 5 by an exhaustive analysis of a network consisting of five agents. Finally, the paper's conclusions appear in Section 6.

2. PRELIMINARIES

The following notation will be used through the paper: I_n denotes the $n \times n$ identity matrix; A^T and $\boldsymbol{a}^T$ are, correspondingly, the transpose of matrix A and the transpose of column vector $\boldsymbol{a} = [a_1, \ldots, a_n]^T$; $\mathcal{S}(A) = \{\lambda_1(A), \lambda_2(A), \ldots, \lambda_n(A)\}$ denotes the spectrum of matrix $A = A^T \in \mathbb{R}^{n\times n}$, where for a real spectrum of A the eigenvalues, $\lambda_i(A)$ for $i = 1, \ldots, n$, are indexed in decreasing order; $A \otimes B$ denotes the Kronecker product of $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$.

Definition 1. A matrix $A \in \mathbb{R}^{n\times n}$ is called *stable* or *Hurwitz* if all its eigenvalues have negative real part, i.e. $\mathcal{S}(A) \subseteq \mathbb{C}_{-}$.

Definition 2. Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$. The matrix A is *similar* to B if there is an invertible matrix $P \in \mathbb{R}^{n\times n}$, such that $A = P^{-1}BP$.

Next, we present some concepts and basic results on graph theory, which are necessary for the development of the paper.

A multi-agent system is represented by an *undirected* graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes (or vertices), $\mathcal{V} = \{1, 2, \ldots, N\}$, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, j \neq i\}$. Communication between agents is bidirectional and agents i and j are said to be neighbours if $(i, j) \in \mathcal{E}$. Each node has associated *degree* or *valency*, d_i for $i = 1, 2, \ldots, N$, which represents the number of neighbours of agent i. Any undirected graph can be represented by its *adjacency matrix*, $\mathbf{A}(\mathcal{G})$. Let $\mathbf{A}_{i,j} \in \mathbb{R}$ be the (i, j) element of adjacency matrix. Then $\mathbf{A}_{i,i} = 0$ for $i = 1, 2, \ldots, N$ as we assume that there is no edge from node to itself, and

$$\mathbf{A}_{i,j} = \begin{cases} 0 \ \ if \ \ (i, j) \notin \mathcal{E} \ \ \forall i, j = 1, 2, \ldots, N, \ \ i \neq j, \\ 1 \ \ if \ \ (i, j) \in \mathcal{E} \ \ \forall i, j = 1, 2, \ldots, N, \ \ i \neq j. \end{cases}$$

An undirected graph is said to be *complete* if there is an edge between every pair of nodes. Then all nodes will have the same degree, $d = N - 1$, where N is the number of nodes (agents).

Definition 3. (Borrelli and Keviczky (2008)) The class of matrices denoted as $\mathcal{K}^{N_d}_{n,m}(\mathcal{G})$ is defined as

$$\begin{aligned}\mathcal{K}^{N_d}_{n,m}(\mathcal{G}) = &\{M \in \mathbb{R}^{nN_d\times mN_d} \mid M_{ij} = 0 \ \text{ if } \ (i, j) \notin \mathbf{A}, \\ &(M_{ij} = M[(i-1)n + 1 : in, (j-1)m + 1 : jm] \\ &\text{if } \ (i, j) \in \mathbf{A} \ \text{ where } \ i, j = 1, 2, \ldots, N_d\}.\end{aligned}$$

3. PROBLEM FORMULATION

3.1 Centralized LQR Control of Multi-Agent Systems

Let consider the linear continuous-time system of ith agent whose dynamics can be described as

$$\dot{\boldsymbol{x}}_{\boldsymbol{i}}(t) = A\boldsymbol{x}_{\boldsymbol{i}} + B\boldsymbol{u}_{\boldsymbol{i}}, \ \ \boldsymbol{x}(0) = \boldsymbol{x}_{\boldsymbol{i_0}} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $\boldsymbol{x}_{\boldsymbol{i}}(t) \in \mathbb{R}^n$, $\boldsymbol{u}_{\boldsymbol{i}}(t) \in \mathbb{R}^m$ are the state and input vectors of the system at time t, respectively. Then, the collective dynamics of N identical and decoupled systems, indexed as $1, 2, \ldots, N$, is given by

$$\dot{x}(t) = A_a\boldsymbol{x} + B_a\boldsymbol{u}, \ \ \boldsymbol{x}(0) = \boldsymbol{x_0} \tag{2}$$

where the column vectors $\boldsymbol{x}(t) = [\boldsymbol{x}^T_{\boldsymbol{1}}(t), \ldots, \boldsymbol{x}^T_{\boldsymbol{N}}(t)]^T$ and $\boldsymbol{u}(t) = [\boldsymbol{u}^T_{\boldsymbol{1}}(t), \ldots, \boldsymbol{u}^T_{\boldsymbol{N}}(t)]^T$ collect the states and inputs of the N systems, while $A_a = I_N \otimes A$ and $B_a = I_N \otimes B$, with A and B defined as in (1).

The LQR problem for the system (2) is described through the cost function of the form

$$J\big(\boldsymbol{u}(t), \boldsymbol{x_0}\big) = \int_0^\infty \Big(\sum_{i=1}^N \big(\boldsymbol{x_i}(t)^T Q_{ii}\boldsymbol{x_i}(t) + \boldsymbol{u_i}(t)^T R_{ii}\boldsymbol{u_i}(t)\big) + \sum_{i=1}^N \sum_{\substack{j=1 \\ j>i}}^N \Big(\big(\boldsymbol{x_i}(t) - \boldsymbol{x_j}(t)\big)^T Q_{ij}\big(\boldsymbol{x_i}(t) - \boldsymbol{x_j}(t)\big)\Big)\Big)\, \mathrm{d}t.$$

In the more compact notation the LQR cost is defined as:

$$J\big(\boldsymbol{u}(t), \boldsymbol{x_0}\big) = \int_0^\infty \big(\boldsymbol{x}(t)^T Q_a\boldsymbol{x}(t) + \boldsymbol{u}(t)^T R_a\boldsymbol{u}(t)\big)\, \mathrm{d}t \tag{3}$$

where the matrices Q_a and R_a have the following structure:

$$Q_a = \begin{pmatrix} Q_{a_{11}} & Q_{a_{12}} & \ldots & Q_{a_{1N}} \\ \vdots & \vdots & \ddots & \vdots \\ Q_{a_{N1}} & Q_{a_{N2}} & \ldots & Q_{a_{NN}} \end{pmatrix}, R_a = I_N \otimes R, \tag{4}$$

with $Q_{a_{ii}} = \sum_{k=1}^N Q_{ik}$ for $i = 1, \ldots, N$; $Q_{a_{ij}} = -Q_{ij}$ for $i, j = 1, \ldots, N, \ \ i \neq j$; $Q_{ii} = Q^T_{ii} \geq 0$ and $R_{ii} = R^T_{ii} > 0$ for $\forall i$, $Q_{ij} = Q^T_{ij} = Q_{ji} \geq 0$ for $\forall i \neq j$.

We are assuming that the pairs (A, B), (A_a, B_a) are stabilizable and the pairs (A, C), (A_a, C_a) are observable for any $Q = Q^T \geq 0$ and Q_a as in (4) (where $C^T C = Q$,