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Auxiliary dynamics for observer design of nonlinear TS systems with unmeasurable premise variables

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Abstract: In this paper, the problem of observers for Takagi-Sugeno (TS) models with unmeasurable premise variables is investigated and a new design approach is proposed. The idea, is motivated by the immersion techniques and auxiliary dynamics generation, and consists in the transformation of the TS system with state dependent weighting functions in a new TS system with weighting functions depending only on the measured variables. This result aims to relax the strong conditions used in the design of observers for TS systems with unmeasurable premise variables. Illustrative example is provided to discuss the performances of the proposed approach.

Keywords: Nonlinear Systems, Takagi-Sugeno Systems, Unmeasurable Premise Variables, Dynamic state extension.

1. INTRODUCTION

From the beginning of the years 2000, the problem of state observer design for Takagi-Sugeno systems having state dependent weighting functions was investigated. Until now, it remains an open problem. The focus is made on the case where the states involved in the weighting functions are not known. The first result was proposed in Bergsten et al. [2001] which extends the Thau-Luenerger observer Thau [1973]. This result has provided LMI conditions depending on the assumption that the perturbation-like term is Lipschitz. The LMIs depend on the Lipschitz constant. This result is extended in Lendek et al. [2009] for TS cascaded systems and in Ichalal et al. [2010] by splitting the perturbation-like term and compute the Lipschitz constants of the weighting functions. The main drawback of these approaches is in the Lipschitz constant LMI-dependent. Indeed, the admissible Lipschitz constant (maximal value of this constant) allowing the existence of a solution to the LMI conditions is often very small which limits the applicability of the Lipschitz approach. Inspired by the differential mean value theorem, used for Lipschitz nonlinear systems Zemouche et al. [2008], a new result is provided by applying this theorem for TS systems with unmeasurable premise variables in Ichalal et al. [2011b], Ichalal et al. [2011a]. The interest of such a result is that it has proposed LMI conditions free from the Lipschitz constant and with asymptotic convergence property. Then, this approach overcomes the limitation related to the Lipschitz assumption and its constant. However, the number of LMIs may become huge which introduce computational complexity. Recently, works have been proposed in order to reduce the conservatism of the Lipschitz approach. The idea is to leave the asymptotic convergence for only bounded error convergence, see for example the quasi Input-to-State Stability (qISS) approach in Ichalal et al. [2012].

Generally, since the linear systems domain is well understood and huge number of algorithms and theories have been developed for control, observation and analysis, the central questions that arise from the beginning of the nonlinear systems story are: Are there coordinate transformations that transform the original system into a linear one in the new coordinates? What are the conditions ensuring the existence of such coordinate transformations? Due to recent developments of some classes of nonlinear systems the first question is relaxed by seeking coordinate transformations that transform the system in nonlinear particular structures, for example, linear systems modulo output injection or state affine systems. In the TS framework, until now, the only used transformation is the sector nonlinearity transformation. Indeed, in the major part of control and observer design, the nonlinear system is kept in the original coordinates and transformed it in a polytopic form (TS form). Especially, in the state observation field, this reasoning introduced several difficulties in stability study of the state estimation error when the premise variables depend of the unmeasured states of the system.

This paper is motivated by the notion of exact linearization without feedback of nonlinear systems introduced in Krener and Isidori [1983] which has aroused great interest as shown by the rich literature in this domain Kazantzis and Kravaris [1998], Glumineau et al. [1996], Keller [1987], Besancon [2007], Souleiman et al. [2003], etc. It introduces a new algorithm for state observer design for TS systems

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by using the immersion techniques which transforms a TS system with unmeasurable premise variables into a TS system with measurable premise variables in the new coordinates. The proposed immersion algorithm can be viewed as an extension of the state vector with new variables (auxiliary dynamics). Notice that using TS structure reduces the complexity in searching an adequate coordinate transformation compared to the existing approaches which seek of new coordinate transformations in order to have a special nonlinear structure (observable normal forms). In the TS framework, the only objective is to find a LPV system with parameters depending on measured signals, then the LPV system can be transformed into a TS form by using the sector nonlinear transformation in a compact set of the state space.

2. PRELIMINARIES AND PROBLEM STATEMENT

Let us consider the nonlinear system

$$\dot{x}(t) = f(x(t), u(t)), \quad y(t) = h(x(t)) \tag{1}$$

In the context of Takagi-Sugeno systems, the system (1) can be transformed into a T-S system by using the sector nonlinearity approach Tanaka and Wang [2001] in a compact set of the state space as follows

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{r} h_i(\xi(t)) C_i x(t) \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_y}$, $y \in \mathbb{R}^{n_y}$ and A_i, B_i and C_i are known matrices with appropriate dimensions. The weighting functions $h_i(\xi(t))$ depend on the premise variable $\xi(t)$ and satisfy the convex sum property. Firstly, assume that the premise variable $\xi(t)$ depend only on measured signals or signals available at real time. Consequently, the state observer takes the form

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\xi(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(\xi(t)) C_i \hat{x}(t) \end{cases} \tag{3}$$

Note that the TS system (2) and the observer (3) share the same premise variables. By defining the state estimation error $e(t) = x(t) - \hat{x}(t)$, its dynamics obeys to the differential equation

$$\dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\xi(t)) h_j(\xi(t)) \left(A_i - L_i C_j\right) e(t) \tag{4}$$

Notice that the stability study of the TS systems of the form (4) has attracted a lot of attention and several approaches were proposed in order to provide less conservatism LMI conditions Tanaka and Wang [2001], Guerra et al. [2006], Sala and Arinõ [2007] etc.

However, in the case of premise variables depending on unmeasured states $\xi(t) = x(t)$, these approaches cannot be directly exploited. Indeed, in such a case, the observer does not share the same premise variables as the TS system (2) but only the estimated ones as follows

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(\hat{\xi}(t)) \left(A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\right) \\ \hat{y}(t) = \sum_{i=1}^{r} h_i(\xi(t)) C_i \hat{x}(t) \end{cases} \tag{5}$$

All the techniques dealing with this case considers linear output equation $y(t) = Cx(t)$. With this restriction, the state estimation error dynamics becomes

$$\dot{e}(t) = \sum_{i=1}^{r} h_i(\hat{\xi}(t)) \left(A_i - L_i C\right) e(t) + \delta(t) \tag{6}$$

where $\delta(t) = \sum_{i=1}^{2} \left(h_i(\xi(t)) - h_i(\hat{\xi}(t))\right) (A_i x(t) + B_i u(t))$. It is then clear that studying the stability of the system (6) generating the state estimation error is more difficult than that of the system (4). Intensive researches have been devoted to this problem and some results were provided. In Bergsten et al. [2001], a method based on the Lipschitz assumption of the term $\delta(t)$ has been established which ensures asymptotic convergence of the state estimation error toward zero. The same idea were used in Lendek et al. [2009] and Ichalal et al. [2010]. The main drawback of these approaches is that the LMI conditions are feasible only for Lipschitz constants having very small values which limits the domain of applicability of these approaches. In addition, the computation of the Lipschitz constant may become a hard task for complex systems. In order to avoid these problems, an approach based on the mean real value theorem and the sector nonlinearity transformation Ichalal et al. [2011b],Ichalal et al. [2011a]. The main advantage of this approach is the establishment of LMI conditions free form the Lipschitz constant. However, For complex systems it may happen that the number of LMIs becomes huge which limits the existence of a solution, nevertheless, the limitation related to the Lipschitz constant is avoided. Notice that the cited approaches aim to provide conditions ensuring asymptotic or exponential convergence. More recently, new approaches have been proposed by replacing the asymptotic convergence by only the bounded error convergence. It is illustrated that the LMI conditions are relaxed compared the asymptotic approaches, hence, the bounded error property is the price to pay to obtain a solution (see for example, Ichalal et al. [2012] by using the $\mathcal{L}_2$ approach or quasi-Input-to-State Stability (qISS)).

In this paper, a new idea is proposed. it is the answer to the question: Is there a coordinate transformation aiming to transform the TS system with unmeasurable premise variables into a new TS system having weighting functions depending only on measured variables? The present paper addresses an answer based on the immersion techniques and auxiliary dynamics used in some observation and control problems of nonlinear systems. The approach is: from a nonlinear system of the form (1) and before using the sector nonlinear transformation, the auxiliary dynamics are used in order to express the system in a Linear Parameter Varying (LPV) system where the matrices depend only on measured variables. Then, the sector nonlinearity transformation can be used in order to express exactly the new system in TS form. Finally, since the premise variables of the obtained TS model are