

# Hidden attractors in electromechanical systems with and without equilibria

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**Abstract:** This paper studies hidden oscillations appearing in electromechanical systems with and without equilibria. Three different systems with such effects are considered: translational oscillator-rotational actuator, drilling system actuated by a DC-motor and drilling system actuated by induction motor. We demonstrate that three systems experience hidden oscillations in sense of mathematical definition. While some of these hidden oscillations can be easily seen in natural physical experiments, the localization of others requires special efforts.

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## 1. INTRODUCTION

The study of stability and oscillations in electromechanical systems requires the construction of mathematical model and its analysis. In addition to normal operation mode the system may experience unwanted oscillations which lead to its failure. Finding the basin of attraction of such oscillations can be a challenging task. Depending on simplicity of finding the basin of attraction in the phase space it is natural to suggest the following classification of attractors (Kuznetsov et al., 2010; Leonov et al., 2011, 2012; Leonov and Kuznetsov, 2013; Kuznetsov, 2016): *An attractor is called a hidden attractor if its basin of attraction does not intersect with small neighborhoods of equilibria, otherwise it is called a self-excited attractor.* Self-excited attractor's basin of attraction is connected with an unstable equilibrium. Therefore, self-excited attractors can be localized numerically by the *standard computational procedure* in which a trajectory, which starts from a point of an unstable manifold in a neighbourhood of an unstable equilibrium, after a transient process is attracted to the state of oscillation (i.e. to an attractor) and traces it. In contrast, hidden attractor's basin of attraction is not connected with unstable equilibria. For example, hidden attractors are attractors in the systems with no equilibria or with only one stable equilibrium (a special case of multistable systems and coexistence of attractors). Recent examples of hidden attractors can be found in *The European Physical Journal Special Topics "Multistability: Uncovering Hidden Attractors"*, 2015 (Leonov et al., 2015b; Shahzad et al., 2015; Brezetskyi et al., 2015; Jafari et al., 2015; Zhusubaliyev et al., 2015; Saha et al., 2015; Semenov et al., 2015; Feng and Wei, 2015; Li et al., 2015; Feng et al., 2015; Sprott, 2015; Pham et al., 2015; Vaidyanathan et al., 2015; Sharma et al., 2015)).

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Hidden oscillations appear naturally in systems without equilibria, describing various mechanical and electromechanical models with rotation. One of the first examples of such models was described by Arnold Sommerfeld in 1902 (Sommerfeld, 1902). He studied vibrations caused by a motor driving an unbalanced mass and discovered the resonance capture (Sommerfeld effect). The Sommerfeld effect represents the failure of a rotating mechanical system to be spun up by a torque-limited rotor to a desired rotational velocity due to its resonant interaction with another part of the system (Evan-Iwanowski, 1976; Eckert, 2013). Relating this phenomenon to the real world Sommerfeld wrote, "This experiment corresponds roughly to the case in which a factory owner has a machine set on a poor foundation running at 30 horsepower. He achieves an effective level of just 1/3, however, because only 10 horsepower are doing useful work, while 20 horsepower are transferred to the foundational masonry" (Eckert, 2013).

We consider three different systems, which have multi stability and experience hidden oscillations in sense of mathematical definition. At the same time we will show that some of these oscillations can be localized if physical nature of the process in such systems is taken into account.

## 2. TRANSLATIONAL OSCILLATOR-ROTATIONAL ACTUATOR

Following the works (Evan-Iwanowski, 1976; Fradkov et al., 2011) we consider the electromechanical "translational oscillator-rotational actuator" (TORA) system (see Fig. 1). It consists of DC motor which actuates the eccentric mass  $m$  with eccentricity  $l$  connected with the cart  $M$ . The cart is elastically connected to the wall with help of a string and moves only horizontally. The equations of the system are the following

$$\begin{aligned} (M + m)\ddot{x} + k_1\dot{x} + ml(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + kx &= 0, \\ J\ddot{\theta} + k_\theta\dot{\theta} + ml\ddot{x} \cos \theta &= u, \end{aligned} \quad (1)$$

Here  $\theta$  is rotational angle of the rotor,  $x$  is the displacement of the cart from its equilibrium position,  $u$  is motor torque,  $k$  is a stiffness of the string,  $k_1$  and  $k_\theta$  are damping coefficients,  $I$  is a moment of inertia.

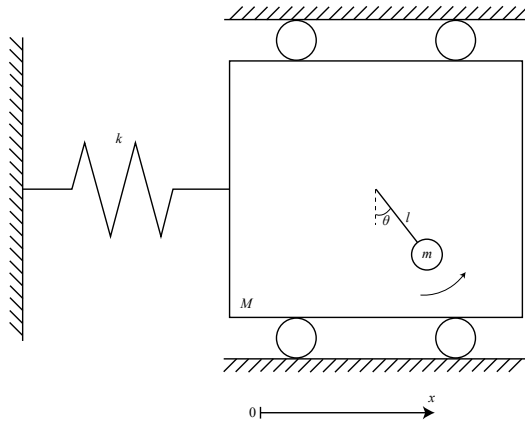


Fig. 1. Translational oscillator-rotational actuator scheme

Note that for  $u \neq 0$  this system has no equilibria. Consider the following parameters of the system (Fradkov et al., 2011):  $J = 0.014$ ,  $M = 10.5$ ,  $m_0 = 1.5$ ,  $l = 0.04$ ,  $k_\theta = 0.005$ ,  $k = 5300$ ,  $k_1 = 5$ . For  $u = 0.48$  the system experiences co-existence of attractors (i.e. multistability). The first attractor corresponds to Sommerfeld effect and it may be observed for initial data  $\dot{x} = x = \theta = \dot{\theta} = 0$  (zero initial data represent typical start of the system, this effect can be easily found). For other initial data  $\dot{x} = x = \theta = 0, \dot{\theta} = 40$  we observe another attractor which is normal operation – the achievement of desired rotational velocity of our mechanical system.<sup>1</sup> In Fig. 2 the transient process for both initial data is shown, in Fig. 3 we observe the attractors, which are obtained after the transient process. All numerical results in this article are obtained with the help of Matlab. Note that if we compare this result with the experiment of Sommerfeld, we see that an effective level of about 1/4 (comparing to normal operation) is achieved here when Sommerfeld effect occurs.

### 3. DRILLING SYSTEMS

Consider now another electromechanical system – drilling system. Drilling systems are widely used in oil and gas industry for drilling wells. The failures of drilling systems cause considerable time and expenditure loss for drilling companies, so the understanding of these failures is a very important task. Here we consider two mathematical models of drilling systems and study their behaviour after operation start. For drilling systems two different ways of operation start are possible: no-load start and start with load. No-load start means that at initial moment of time there is no friction torque acting on the lower disc. The start with load is start of the drilling with friction torque acting on the lower disc at initial moment of time (this case also corresponds to a sudden change of rock type).

<sup>1</sup> Both effects were modelled in (Fradkov et al., 2011), but in our work we give the information on parameters more accurate

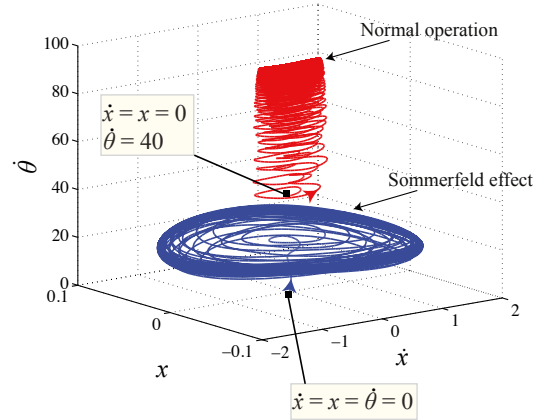


Fig. 2. Sommerfeld effect and normal operation in TORA

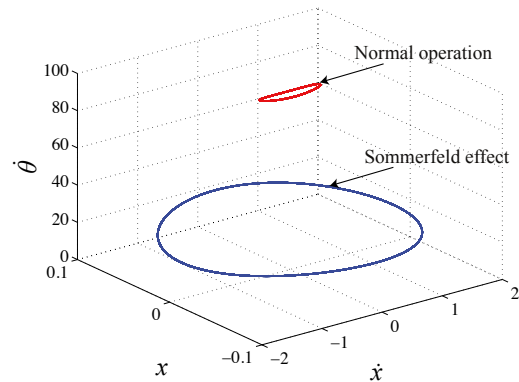


Fig. 3. Sommerfeld effect and normal operation in TORA – localization after transient process

#### 3.1 Drilling system actuated by DC motor

In the (de Bruin et al., 2009; Mihajlovic et al., 2004) works the scientific group from Eindhoven University of Technology constructed and studied an experimental setup which consists of two discs connected with a steel string. The upper disc represents the rotary table of the drilling system and is actuated by a DC-motor (see schematic view of the system in Fig. 6). The lower disc represents bottom hole assembly.

For the construction of mathematical model of the system it is assumed that the drill string is massless and experiences only torsional deformation. The system is described by the following equations:

$$\begin{aligned} J_u \ddot{\theta}_u + k_\theta (\theta_u - \theta_l) + b (\dot{\theta}_u - \dot{\theta}_l) + T_{fu} (\dot{\theta}_u) \\ - k_m v &= 0, \\ J_l \ddot{\theta}_l - k_\theta (\theta_u - \theta_l) - b (\dot{\theta}_u - \dot{\theta}_l) + T_{fl} (\dot{\theta}_l) &= 0, \end{aligned} \quad (2)$$

where  $\theta_u(t)$  and  $\theta_l(t)$  are angular displacements of the upper and lower discs with respect to the earth,  $J_u$  and  $J_l$  are constant inertia torques,  $b$  is rotational friction,  $k_\theta$  is the torsional spring stiffness,  $k_m$  is motor constant,  $v$  is constant input voltage.  $T_{fu}$  and  $T_{fl}$  are friction torques acting on the upper and on the lower disc, respectively.

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