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Synchronization of Heteroclinic Circuits Through Learning in Chains of Neural Motifs ⋆

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Abstract: The synchronization of oscillatory activity in networks of neural networks is usually implemented through coupling the state variables describing neuronal dynamics. In this study we discuss another but complementary mechanism based on a learning process with memory. A driver network motif, acting as a teacher, exhibits winner-less competition (WLC) dynamics, while a driven motif, a learner, tunes its internal couplings according to the oscillations observed in the teacher. We show that under appropriate training the learner motif can dynamically copy the coupling pattern of the teacher and thus synchronize oscillations with the teacher. Then, we demonstrate that the replication of the WLC dynamics occurs for intermediate memory lengths only. In a unidirectional chain of N motifs coupled through teacher-learner paradigm the time interval required for pattern replication grows linearly with the chain size, hence the learning process does not blow up and at the end we observe phase synchronized oscillations along the chain. We also show that in a learning chain closed into a ring the network motifs come to a consensus, i.e. to a state with the same connectivity pattern corresponding to the mean initial pattern averaged over all network motifs.

Keywords: Learning, Synchronization, Network motifs, Nonlinear dynamics.

1. INTRODUCTION

Complex large-size biological, ecological, and engineering networks can be frequently decomposed into relatively small network motifs, i.e. network patterns that occur significantly more frequently than in a random graph. Then, the study of the network structural properties can be addressed through investigation of universal classes or building blocks of recurrent network motifs (Milo et al., 2002). How one network motif can dynamically replicate the internal structure and the behavior of another one is an open problem.

Traditionally, synchronization of oscillations in network systems involves transmission of signals (energy) from one network element to another. For example, in neural networks synaptic couplings may convey electrical or chemical signals from one motif to another, which frequently promotes synchronization (Abarbanel et al., 1996). However, synchronization can also be achieved through a learning process. In this case there is no direct link between two networks. Instead, the information transfer is attained through observation of the teacher dynamics and by consecutive tuning of the connectivity pattern in the learner. Although such kind of synchronization is abundant in real world (e.g. children can learn movements shown by a teacher), its study from a dynamical systems point of view has attracted relatively little attention.

Earlier it has been shown that oscillations in network systems can emerge from a stable heteroclinic channel (Ashwin and Chossat, 1998; Ashwin and Field, 1999). In a neural network consisting of more than two competing neurons with unbalanced inhibitory connections, one may observe a situation when each neuron sequentially becomes a winner (i.e. strongly activated) for a limited time interval and then another neuron takes over the leadership. Dynamically such an operating mode, called winner-less competition (WLC), occurs in a vicinity of heteroclinic trajectories connecting saddle equilibria in a loop. Under certain conditions, the heteroclinic loop can be stable and then in the presence of a weak noise the trajectory will wander from one saddle to another (Cohen and Grossberg, 1983; Rabinovich et al., 2001; Varona et al., 2002).

In this work we propose a learning rule which allows one neural network, acting as a teacher, to impose the same heteroclinic circuit in another "learner" network. As a result, in the learner there appear WLC oscillations synchronized in phase with the oscillations of the teacher. We study how the information on the connectivity structure is replicated in a chain of network motifs. The proposed learning rule includes memory effects, i.e. the learner integrates over time the incoming information. We

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then provide conditions necessary for replication of the connectivity patterns in a learning chain of network motifs and also describe a “consensus” behavior on a ring.

2. MODEL DYNAMICS: SYNCHRONIZATION BY LEARNING

Figure 1A shows the architecture of a network motif composed of three recurrently coupled neurons. For the sake of simplicity we assume that the coupling strengths β are hard coded (i.e. fixed), while $\alpha = \{\alpha_k\}_{k=1}^3$ can be changed. Further on we will consider a unidirectional chain of such motifs (Fig. 1A). At the beginning the couplings α are arbitrary distributed among network motifs, and hence the motifs exhibit different dynamics. The purpose of learning is to replicate the coupling pattern α from the teacher and synchronize oscillations along the chain.

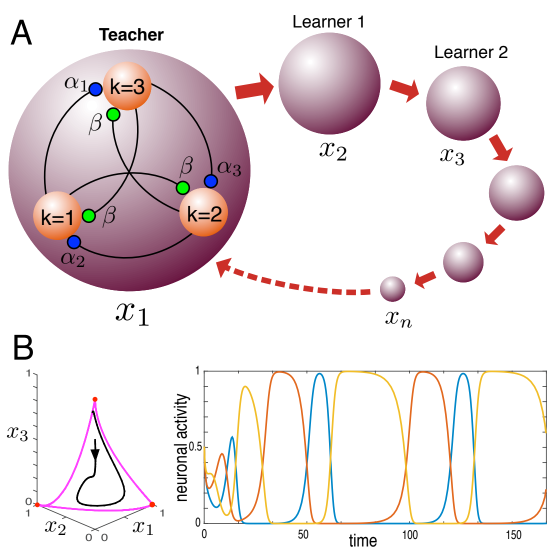

Fig. 1. The model. A) Network motifs consist of three recurrently coupled neurons each. Motifs are linked in a unidirectional learning chain. No direct coupling among the state variables exists. Instead, learner n adjusts its connectivity pattern to that of motif $n-1$ and thus synchronizes oscillations. If the last motif is linked to the first one we get a ring chain without leader. B) Winner-less dynamics in the phase space of a single motif (left) and time evolution of the neuronal activity (right). Blue, red, and yellow curves correspond to neurons 1, 2, and 3, respectively [$\alpha = (0.1, 0.6, 0.8)$].

2.1 Heteroclinic circuit: Winner-less dynamics

The network of motifs is organized by the teacher-learner principle. The governing equation of the teacher is given by the generalized Lotka-Volterra system

$$\dot{x} = x \odot (1 - \rho x) + \eta(t) \qquad (1)$$

where $x(t) \in \mathbb{R}^3_+$ describes the activation state of three neurons at time t (Fig. 1B); $\odot$ stands for the Hadamard product; $\eta(t) \in \mathbb{R}^3$ is a Gaussian uncorrelated white noise with the mean 2e-5 and the standard deviation 1.5e-3; and the matrix $\rho \in \mathcal{M}_{3\times 3}(\mathbb{R}_+)$ accounts for local couplings among the neurons:

$$\rho = \begin{pmatrix} 1 & \alpha_2 & \beta \\ \beta & 1 & \alpha_3 \\ \alpha_1 & \beta & 1 \end{pmatrix}$$

Given that the following conditions are satisfied

$$\alpha_k < 1 < \beta, \quad \prod_{k=1}^{3}(1-\alpha_k) < (\beta - 1)^3, \qquad (2)$$

it has been shown (Afraimovich et al., 2004) that in the system (1) there exists a globally stable heteroclinic circuit (Fig. 1B, left). Further on we will assume that $\beta > 2$ ($\beta = 2.8$ in numerical simulations). Then, condition (2) will be satisfied for any $\alpha_k < 1$. We will use double notation: index k will be used for all intra-motif variables, whereas index n will denote the motif number in the learning chain.

2.2 Synchronization by learning

Let us now consider a learning chain of network motifs (Fig. 1A). Since the learning is unidirectional, we can consider a pair teacher-learner, i.e. motifs n and $n-1$ will be referred to as a learner and a teacher, respectively.

Under phase synchronization by learning we understand the situation when independently on the teacher coupling pattern α_{n-1} and the initial conditions $[x_{n-1}(0),\ x_n(0),\ \alpha_n(0)]$ after some transient the following inequality is satisfied

$$|\phi_{n-1}(t) - \phi_n(t)| < M \qquad (3)$$

where ϕ_{n-1} and ϕ_n are the oscillatory phases in the teacher and in the learner, respectively, and M is a constant.

Without loss of generality, we can assume that each teacher motif has a fixed coupling structure. At the beginning, the connectivity pattern in the learner $\alpha_n(0)$ is taken arbitrary from uniform distribution over $(0,1)^3$. Then the purpose of learning is to “copy” the coupling structure and consequently to synchronize oscillations in the learner with the teacher.

Since the teacher network cannot change the learner state $x_n(t)$ directly, but through the coupling strengths $\alpha_n(t)$ only, during the learning we expect:

$$\lim_{t\to\infty} \|\langle \alpha_n \rangle_T(t) - \alpha_{n-1}\|_2 = 0 \qquad (4)$$

where

$$\langle u \rangle_T(t) = \frac{1}{T}\int_{t-T}^{t} u(s)\,\mathrm{d}s$$

denotes the time averaging operator over period T. Then, fulfillment of (4) ensures (3). In numerical simulations the learning will be deemed finished if the norm in (4) falls below a tolerance value $0 < \delta \ll 1$ for some t^*.

2.3 Learning rule

We will employ a Hebb-like rule for learning. First, we introduce a functional:

$$g(u(t)) = u(t) \odot \frac{1}{\tau}\int_{t-\tau}^{t} u(s)\,\mathrm{d}s \qquad (5)$$

where $\tau \geq 0$ is a constant describing the memory length. The function $g(x_{n-1})$ represents the cumulative activity