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Controlled synchronization in two hybrid FitzHugh-Nagumo systems

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Abstract: We study synchronization in two FitzHugh-Nagumo systems with discrete coupling, which are the simplest model of neural network. It is well known that high delays in propagation between the nodes hinder synchronization. We use the linear matrix inequality method to study the impact of the discretization step on the system synchronization. We show that external stimulus can be used for controlling synchrony in the case of its absence. We develop the algorithm for synchronization of FitzHugh-Nagumo systems and find the conditions of its applicability. The simulation results confirm the efficiency of suggested algorithm.

Keywords: Hybrid models, Oscillation, Neural dynamics, Synchronization control.

1. INTRODUCTION

Synchronization phenomenon attracts a lot of attention of researchers from different scientific communities for many years Blekhman (1988); Pikovsky et al. (2001). An area of special interest is synchronization in large populations of interacting oscillatory elements Winfree (1980); Kuramoto (1984); Tass (1999); Strogatz (2014); Schöll et al. (2016), relevant to many problems of physics, biology, chemistry and engineering, in particular, to neuroscince.

As any other kind of physical, chemical, or biological oscillators, such neurons could synchronize and exhibit collective behavior that is not intrinsic to any individual neuron. For example, partial synchrony in cortical systems is believed to generate various brain oscillations, such as the alpha and gamma EEG rhythms. Coordinated synchrony is needed for locomotion and swim pattern generation in fish Izhikevich (2005). On the one hand, synchronization can be good, while, on the other hand, it can be harmful. For example, synchronization of individual neurons is believed to play the crucial role in the emergence of pathological rhythmic brain activity in Parkinson's disease, essential tremor, and epilepsies; a detailed discussion of this topic and numerous citations can be found in Refs. Tass (1999); Golomb et al. (2001).

In order to grasp the complicated interaction of neurons in large neural networks, those are often lumped into groups of neural populations each of which can be represented as en effective excitable element that is mutually coupled to other elements Rosenblum and Pikovsky (2004); Popovych et al. (2004). In this sense the simplest model which may reveal features of interacting neurons consists of two coupled neural oscillators. Each of this can be represented by a simplifiled FitzHugh-Nagumo system FitzHugh (1961); Nagumo et al. (1962). In recent papers Plotnikov (2015); Plotnikov (2015) we investigate the synchronization in two FitzHugh-Nagumo networks with heterogeneous thresholds and slowly-varying delays in signal propagation, respectively. Here we focus on the systems with discrete coupling, because the neuron affects on its neighbors by spiking, which occurs between the resting periods.

The problem of the estimation of the discretization step (interval) providing stability and acceptable system performance is nontrivial. Some results for linear systems were obtained in Fridman (2010), while the method of discretization step estimation was recently generalized for nonlinear systems Seifullaev and Fradkov (2015). In the recent years the international literature manifested interest in a new approach based on the rearranging the discrete-continuous model of the system to the form with varying (sawtooth) delay. Using this representation the methods of Lyapunov-Krasovskii functionals and Lyapunov-Razumikhin functionals find the wide implementation Kharitonov and Zhabko (2003); Gelig and Zuber (2011). Here we use the method suggested in Seifullaev and Fradkov (2015) for the estimation of the discretization step while the system is possible to synchronize, because the considered system is nonlinear. If synchronization of the system without control is impossible, then we use the controller designed by an application of the Halanay's inequality Halanay (1966) that extends the Razumikhin method Gu et al. (2003).

The paper is organized as follows. After this introduction we describe the model system and define the synchronization problem in Sec. 2. Section 3 studies the dependency of the upper boundary value of the discretization step in the coupling on the coupling strength, while in Sec. 4 the control algorithm is described to ensure the systems synchronization. Finally, we conclude with Sec. 5.

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2. MODEL EQUATION

The FitzHugh-Nagumo (FHN) model FitzHugh (1961); Nagumo et al. (1962) consists of a modified version of the Van der Pol's equations Van der Pol (1926, 1927) to describe relaxation oscillators, aiming to capture the characteristics of neuronal oscillations. The motion equations are defined by two state variables representing excitability and refractoriness, the membrane potential u and a recovey variable v, respectively, as described in (1):

$$\begin{aligned} \dot{u} &= u - \frac{u^3}{3} - v, \\ \dot{v} &= \varepsilon(u + a - bv), \end{aligned} \tag{1}$$

where the parameters a, b and ε are constants with values 0.7, 0.8 and 0.1, respectively. This relatively simple dynamical system can reproduce several phenomena related to excitable cells in responce to a stimulus, such as subthreshold oscillations, suprathreshold oscillations (action potentials), relative refractoriness and absolute refractoriness, just to mention a few Schwan (1969); Koch (1999); Izhikevich (2005); Soriano et al. (2012).

Now consider two coupled FHN systems, which are the simplest model of the neural network:

$$\begin{aligned} \dot{u_1} &= u_1 - \frac{u_1^3}{3} - v_1 + C[u_2^c(t) - u_1(t)] + I(t), \\ \dot{v_1} &= \varepsilon(u_1 + a - bv_1), \\ \dot{u_2} &= u_2 - \frac{u_2^3}{3} - v_2 + C[u_1^c(t) - u_2(t)], \\ \dot{v_2} &= \varepsilon(u_2 + a - bv_2), \end{aligned} \tag{2}$$

where C is a coupling strength and $I(t)$ is an external stimulus, which is considered as a control. The propagation signal is assumed to be generated by a zero-order hold function with a sequence of sampling instants $0 = t_0 < t_1 < \dots < t_k < \dots$

$$u_1^c(t) = u_1(t_k), \quad u_2^c(t) = u_2(t_k), \quad t_k \le t < t_{k+1}, \tag{3}$$

where $\lim_{k\to\infty} t_k = \infty$.

The reason to consider the discrete propagation signal is that neuron affects on its neighbors by spiking, which occurs between the resting periods. Assume that the inequalities

$$t_{k+1} - t_k \le h \quad \forall k \ge 0 \tag{4}$$

are satisfied for some $h > 0$. Following Mikheev et al. (1988), we represent the digital signal as a delayed signal as follows:

$$\begin{aligned} u_i^c(t) = u_i(t_k) = u_i(t - \tau(t)),\ i = 1, 2 \\ \tau(t) = t - t_k,\ t_k \le t < t_{k+1}. \end{aligned} \tag{5}$$

Let us formulate the problem of variable value synchronization in two coupled FHN systems. To this end subtract the third equation from the first one, and the fourth one from the second one (2), respectively, making the following substitution

$$\delta_1 = u_1 - u_2, \qquad \delta_2 = v_1 - v_2, \tag{6}$$

and obtain

$$\begin{aligned} \dot{\delta}_1 &= (1 - C - \phi)\delta_1 - C\delta(t - \tau) - \delta_2 + I, \\ \dot{\delta}_2 &= \varepsilon(\delta_1 - b\delta_2), \\ t &\in [t_k,\ t_{k+1}), \qquad \tau(t) = t - t_k. \end{aligned} \tag{7}$$

where $\phi = 1/3(u_1^2 + u_1u_2 + u_2^2)$, $\phi(t) \ge 0$, $\forall t$. Our objective is to study the impact of the discretization step h on stability of the system (7) without control, i.e. $I(t) = 0$, and design the control algorithm to ensure the stability of the system (7) in the case of its absence.

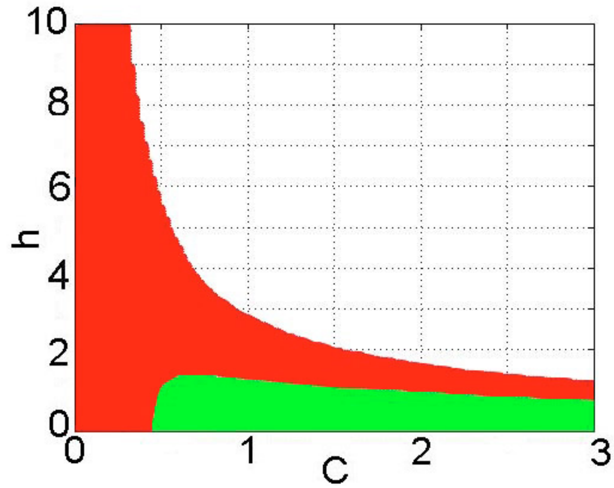

Fig. 1. Solvability of the LMI in *Theorem* 2 Seifullaev and Fradkov (2015) for the system (7) (green color), and area of the system (7) synchronization obtained by the simulation (red color). Parameters: $b = 0.8$, $a = 0.7$, $\varepsilon = 0.1$.

3. STABILITY ANALYSIS

In this section we study the influence of the discretization step h on the system (7) stability depending on the coupling strength C. Here we assume that external stimulus $I(t)$ equals 0.

We apply the approach of Seifullaev and Fradkov (2015) to get the estimation of discretization step h needed for system (7) synchronization. For its application we should find the boundary straight lines between which nonlinearity lies. For this purpose let introduce the following Lyapunov function

$$V(t, \mathbf{x}(t)) = \frac{1}{2}\left[u_1^2 + u_2^2 + \frac{1}{\varepsilon}\left(v_1^2 + v_2^2\right) + C\int_{t-\tau(t)}^{t} (u_1^2(s) + u_2^2(s))\mathrm{d}s\right], \tag{8}$$

where $\mathbf{x} = (u_1, u_2, v_1, v_2)$. Meaning substitutions (5) find its derivative according to the system (2) without control $I(t)$:

$$\begin{aligned} \dot{V}(t, \mathbf{x}(t)) &= -u_1^4/3 + u_1^2 - u_1v_1 + Cu_1u_2(t - \tau(t)) - Cu_1^2 \\ &- u_2^4/3 + u_2^2 - u_2v_2 + Cu_2u_1(t - \tau(t)) - Cu_2^2 \\ &+ u_1v_1 + av_1 - bv_1^2 + u_2v_2 + av_2 - bv_2^2 \\ &+ Cu_1^2/2 + Cu_2^2/2 - Cu_1^2(t - \tau(t))/2 - Cu_2^2(t - \tau(t))/2 \\ &= -C(u_1(t - \tau) - u_2)^2/2 - C(u_2(t - \tau(t) - u_1)^2/2 \\ &- (u_1^2 - 3/2)^2/3 - (u_2^2 - 3/2)^2/3 \\ &- b(v_1 - a/2b)^2 - b(v_2 - a/2b)^2 + 3/2 + a^2/2b. \end{aligned} \tag{9}$$

This Lyapunov function derivative is negative if

$$\frac{(u_i^2 - 3/2)^2}{3} > \frac{3}{2} + \frac{a^2}{2b}, \quad i = 1, 2, \tag{10}$$