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Bearing-Only Control of Leader First Follower Formations ⋆

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Abstract: This paper proposes a bearing-only based control law to stabilize leader-first follower formations consisting of single-integrator modeled agents. For leader-first follower formations, under the proposed control law, the N-agents formation is proved to be almost globally asymptotically stable and locally exponentially stable in $\mathbb{R}^d$. Collision avoidance between the agents in the formation is also analyzed.

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1. INTRODUCTION

Recently, formation control of multi-agent systems has attracted lots of research interests. In formation control problems, a group of agents has to achieve a prescribed geometric formation without a centralized sensing and processing unit. To accomplish the group task, each agent needs to obtain some pieces of information, usually relative sensed variables about formation geometry from a subgroup of members.

Based on sensed and controlled variables, the authors of Oh et al. (2015) classified the formation control problems into several categories, namely position-based, displacement-based, distance-based formation control, and some special schemes. Bearing-only based formation control is a special formation control scheme, in which each agent in the group has only relative bearing measurements with regard to its neighbors. Since bearing information could be obtained from visual sensors (Das et al., 2002; Moshtagh et al., 2009), bearing-only based approach reduces number of sensors in the overall systems. This is obviously an advantage in systems of small-sized UAVs.

While distance-based formation control has been extensively investigated in the literature (Krick et al., 2009; Dörfler and Francis, 2010; Cao et al., 2011; Oh and Ahn, 2011), bearing-only based formation control has yet to be fully developed. *Bearing rigidity* in two-dimensional space has been studied in (Eren, 2012; Franchi and Giordano, 2012). The authors in (Franchi and Giordano, 2012) used the terminology *parallel rigidity* and defined the bearing-constrained rigidity matrix. The rank of the bearing constrained rigidity matrix can be used to check bearing rigidity. In (Eren, 2012), bearing-rigidity as well as a bearing-based Henneberg addition construction was developed for planar formations. Also, Eren (2012) proved that minimally bearing rigid formations with the leader-first follower structure occur only in cyclic formations and proposed a bearing-only control law for nonholonomic agents. Control of bearing-only formations in two-dimensional space were also analyzed in Schoof et al. (2014); Bishop et al. (2011). Recently, Zhao and Zelazo (2015b) developed the concepts of bearing rigidity, infinitesimal bearing rigidity, and rigidity matrix to an arbitrary dimension. they proposed a bearing-only control law for formations modeled by bearing rigid *undirected graphs*. The bearing rigidity-based approach has advantages in analyzing systems which consist of a large number of agents.

Another approach is controlling the subtended angle between two relative bearing vectors. For example, formations of three or four agents were analyzed in (Basiri et al., 2010) and (Bishop, 2011), respectively. However, their approach has difficulties in extending to a larger number of agents. Also in this direction, cyclic formations of N agents were proved to be locally asymptotically stable in (Zhao et al., 2014a,b).

Although bearing-only control has been studied extensively for formations with undirected sensing topologies, there is still a lack of treatment in directed cases. *Bearing persistence* for directed topologies has been recently proposed in (Zhao and Zelazo, 2015a), however, that term is only proved to be a sufficient condition for 2D formations. Moreover, the control law in (Zhao and Zelazo, 2015a) is a relative position control law. A directed bearing-only control strategy for a N-agent system with three stationary leaders were proposed in (Trinh et al., 2014). In that paper, the formation has been proved to be locally asymptotically stable in 2-D space.

This paper focus on a bearing-only based formation control problem in which the formation's topology is modeled by

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directed graphs known as *leader-first follower* (LFF) (Anderson et al., 2007; Yu et al., 2009; Summers et al., 2011). In LFF structures, the position of the leader determines the formation's position, while the distance between the first follower and the leader controls the overall formation scale. Other agents are followers; each follower has to maintain two bearing constraints with its two leaders. We prove that bearing equivalence implies bearing congruence in LFF formations. Then, the desired bearing vector set for a LFF formation is defined. Under some assumptions, the formation satisfying the desired bearing vector set is asymptotically achieved. Moreover, there exists a finite time that the desired formation is exponentially stable. Collision-free is also guaranteed between each follower and its two leaders under the proposed control law.

The rest of this paper is organized as follows. Background in algebraic graph theory, main assumptions and problem formulation are provided in Section 2. The main analysis is given in Section 3. Section 4 provides simulation result of an eight-agent LFF formation and Section 5 contains some concluding remarks.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Background

Consider a system consisting of N autonomous agents in $\mathbb{R}^d, d \geq 2$ which share a same reference frame and all have a single integrator dynamics. Let $\mathbf{p}_i \in \mathbb{R}^d, i \in \{1, ..., N\}$ represent i-th agent's position. We denote the distance between any two agents $1 \leq i \neq j \leq N$ by $d_{ij} = \|\mathbf{p}_j - \mathbf{p}_i\|$. If two agents i and j are not collocated, the unit relative bearing vector from i to j is defined as

$$\mathbf{g}_{ij} := \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|} = \frac{\mathbf{z}_{ij}}{\|\mathbf{z}_{ij}\|} = \frac{\mathbf{z}_{ij}}{d_{ij}}. \quad (1)$$

The bearing vector $\mathbf{g}_{ij}$ contains information of the direction from agent i to agent j. Obviously, $\mathbf{g}_{ij} = -\mathbf{g}_{ji}$ and $\|\mathbf{g}_{ij}\| = 1$. For any bearing vector $\mathbf{g}_{ij}$, we define $\mathbf{P}_{\mathbf{g}_{ij}} = \mathbf{I}_d - \mathbf{g}_{ij}\mathbf{g}_{ij}^T$ as an orthogonal projection matrix from $\mathbb{R}^d$ into the nullspace of span$\{\mathbf{g}_{ij}\}$. Then $\mathbf{P}_{\mathbf{g}_{ij}} = \mathbf{P}_{\mathbf{g}_{ij}}^T$, $\mathbf{P}_{\mathbf{g}_{ij}} = \mathbf{P}_{\mathbf{g}_{ij}}^2$, $\mathbf{P}_{\mathbf{g}_{ij}}$ is positive semi-definite, Null$\{\mathbf{P}_{\mathbf{g}_{ij}}\} =$ span$\{\mathbf{g}_{ij}\}$ and eig$\{\mathbf{P}_{\mathbf{g}_{ij}}\} = \{0, 1, \ldots, 1\}$.

The *information architecture* of a formation is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which is defined as a pair $(\mathcal{V}, \mathcal{E})$, where the vertex set $\mathcal{V}$ represents the agents and the edge set $\mathcal{E}$ represents the set of bearing vectors to be controlled to maintain the formation shape.

A directed edge from i to j, denoted by $(i, j) \in \mathcal{E}$, exists if and only if a bearing vector constraint must be actively maintained from i. We call agent j a neighbor of agent i and denote the neighbor set of i by $\mathcal{N}_i$. The out-going degree of a node i is the number of outgoing edges from i, i.e., equals to $|\mathcal{N}_i|$ (Godsil and Royle, 2001).

A directed path is a sequence of edges (i_1, i_2), (i_2, i_3), ..., (i_{k-1}, i_k) in $\mathcal{E}$, where $i_1, i_2, ..., i_k \in \mathcal{V}$. We refer i_1 and i_k as the start vertex and the end vertex, respectively. A directed cycle is a directed path which has the same start vertex and end vertex. An acyclic directed graph is a directed graph which has no directed cycle.

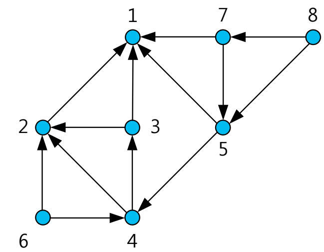

Fig. 1. A LFF graph of 8 nodes. Node 1 (the leader) has out-degree 0, node 2 (the first follower) has out-degree 1, and each other node has out-degree 2.

A formation with information architecture $\mathcal{G}$ at $\mathbf{p}$ is denoted as $\mathbf{F}(\mathbf{p}) = (\mathcal{G}, \mathbf{p})$, where $\mathbf{p} : \mathcal{V} \to \mathbb{R}^{dN}$ is a function which assigns to each vertex a position in the plane, $\mathbf{p} = [\mathbf{p}_1^T, \ldots, \mathbf{p}_N^T]^T$. Two formations $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$ are *bearing equivalent* if $\mathbf{g}_{ij} = \mathbf{g}'_{ij}$ for all $(i, j) \in \mathcal{E}$; they are *bearing congruent* if $\mathbf{g}_{ij} = \mathbf{g}'_{ij}$ for all (i, j) such that $i \neq j, i, j \in \{1, \ldots, N\}$. The bearing congruence between two formations implies the geometrical shapes of two formations in $\mathbb{R}^d$ are similar, or i.e., $\forall 1 \leq i \neq j \leq N$, $d_{ij}/d'_{ij} = c$, where c is a positive constant.

In this paper, we focus on the *leader-first follower* (LFF) formations, or *two-leader formation* as termed in (Eren, 2012). The LFF formation has an underlying graph constructed from a bearing-based Henneberg sequence (Eren, 2012). There is an agent 1 (the leader) with no neighbor. Agent 2 is the first follower, which is supposed to maintain a bearing constraint to the leader. Agent 3 (the second follower) has to maintain two bearing constraints to the leader and the first follower. Similarly, each agent $i(3 \leq i \leq N)$ has to maintain two bearing constraints to two agents $j, k \in \{1, \ldots, i-1\}$. Hence, $\mathcal{G}$ is a minimally acyclic directed graph with $2N-3$ directed edges (Anderson et al., 2008; Hendrickx et al., 2007). For example, an eight-agent LFF formation graph is depicted in Fig. 1.

Definition 1. (Formation scale) For a leader-first follower formation $\mathbf{F}(\mathbf{p}) = (\mathcal{G}, \mathbf{p})$, the formation scale is defined as $\zeta(\mathbf{F}(\mathbf{p})) := \|\mathbf{p}_1 - \mathbf{p}_2\| = d_{21}$.

Theorem 1. Consider a minimally acyclic directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ generated from a Henneberg sequence with more than three vertices ($N \geq 3$). In $\mathbb{R}^d$, if two formations $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$ with the same information structure $\mathcal{G}$ are bearing equivalent, they are also bearing congruent.

Proof. In $\mathbb{R}^d$, consider two bearing equivalent formations $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$ of N agents ($N \geq 3$). We will prove the bearing congruence between $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$ based on mathematical induction of the number of agents in the formation.

Firstly, consider a system of $n = 3$ agents with two bearing equivalent formations $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$. In this case, bearing equivalence implies that $\mathbf{g_{ji}} = \mathbf{g'_{ji}}$, $\forall 1 \leq j < i \leq 3$. From the property of the bearing vector, it follows immediately that $\mathbf{g}_{ij} = \mathbf{g}'_{ij} = -\mathbf{g}_{ji}, 1 \leq i < j \leq 3$ and thus all bearing constraints between $\mathbf{F}(\mathbf{p})$ and $\mathbf{F}(\mathbf{p}')$ are equal. Hence, Theorem 1 is true for $n = 3$. Further, $\forall 1 \leq j \neq i \leq 3$, $d_{ij}/d'_{ij} = \zeta(\mathbf{F}(\mathbf{p}))/\zeta(\mathbf{F}(\mathbf{p}'))$.

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