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Robustness of Controllability for Scale-free Networks Based on a Nonlinear Load-Capacity Model ⋆

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Abstract: Based on a nonlinear load-capacity model, the network evolution and controllability evolution of scale-free networks under the cascading failures triggered by removing the highest-load edge are simulated and discussed in this paper. It is shown by numerical simulations that the controllability evolution is consistent with the average degree evolution rather than the power law exponent evolution. Under the same network cost, it is found that the nonlinear load-capacity model exhibits the stronger robustness of controllability when the initial power law exponent of networks is small by comparing with the linear load-capacity model, while the linear load-capacity model is of stronger robustness of controllability when the initial power law exponent is large. Numerical results shows that high-load edges are becoming more critical to the robustness of controllability with the increase of initial power law exponent, and the nearly highest-load edges are becoming more critical with the increase of both initial average degree and power law exponent.

Keywords: Complex network, controllability, scale-free network, cascading failure.

1. INTRODUCTION

In the past decades, great effort has been devoted to understanding the dynamical processes and the interplay between topology in various complex networks (Albert and Barabási (2002), Newman (2003), Boccaletti *et al.* (2006), Caldarelli (2007), Barabási and Albert (1999), Fortunato (2010), Albert *et al.* (1999), Wang *et al.* (2015), Chen *et al.* (2015a), Chen *et al.* (2015b)). Recently, many researchers have focused on the mechanism of controllability of complex networks. although most real complex networks are driven by nonlinearity, in many aspects, the controllability of various nonlinear systems is structurally similar to the controllability of linear systems (Slotine *et al.* (1991)), which prompts us to study complex networks with canonical linear, time-invariant dynamics. Consider a linear time-invariant system:

$$\dot{x} = Ax + Bu, \tag{1}$$

where vector $x = (x_1, ..., x_N)^T \in R^N$ is the state of system (1) consisting of N nodes. Matrix A is the system's adjacent matrix describing links and interactions. B is the $N \times M$ input matrix ($M \leq N$) that identifies the nodes controlled by external controllers. The input vector $u = (u_1, ..., u_M)^T \in R^M$ imposed by the controllers is used to control system (1). System (1) is said to be controllable if and only if Kalman's matrix $\mathbf{C}$ is full rank, where $\mathbf{C} = (B, AB, A^2B, ...A^{N-1}B)$ (Kalman (1963)). In order to fully control system (1) with a given system matrix A, we should choose an appropriate B (indicates the location of driver nodes) to guarantee the full rank of matrix $\mathbf{C}$. However, for the majority of real networks, it is difficult to obtain exact link weights for matrix A. Besides, the computation of rank($\mathbf{C}$) is also a formidable task for large networks. To bypass the need of measuring the link weights, the structural controllability proposed by Lin (1974) is suitable to overcome our lack of the cognition to the link weights. As we know, system (1) is structurally controllable if it is possible to choose the right non-zero weights in A and B such that matrix $\mathbf{C}$ has full rank. Shields and Pearson (1975) proved that system (1) is controllable for almost all set of link weights of A and B other than a pathological set with zero measure. Further research on exact controllability of any network structure, directed or undirected, with or without link weights and self-loops is studied by Yuan *et al.* (2013), which is based on the maximum multiplicity of eigenvalue.

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Generally speaking, real complex networks contain unreliable components. For example, in critical infrastructure and technological networks, some links may become non-operational due to disasters or attacks (Solé *et al.* (2008)). Although the robustness and resilience of networks have been extensively investigated over the past decades (Motter and Lai (2002), Motter (2004), Wu *et al.* (2006), Newman (2010), Cohen *et al.* (2000), Cohen *et al.* (2001) and Jeong *et al.* (2000)), the robustness of controllability for complex networks has not been sufficiently explored. Wang and Sun (2010), Liu *et al.* (2011), Pu *et al.* (2012), Ruths and Ruths (2013) and Nie *et al.* (2014) investigated the robustness of network controllability under node (or edge) attack and cascading failures based on a linear load-capacity model (ML model) proposed by Motter and Lai (2002). In the ML model, it is simply defined that the capacity is linearly proportional to the load. However, based on the analysis of real networks, Kim and Motter (2008) found that relationship between the load and the capacity shows a nonlinear behavior: the traffic fluctuations on high-load elements is small, which means there is a reduction of unoccupied capacity with the increase of the edge load (smaller load-to-capacity ratio). The nonlinear load-capacity relationship is contrasts with the default assumption used in previous studies.

Therefore, in this paper, we study the controllability robustness of scale-free networks against cascading failures based on a nonlinear load-capacity model. The rest of this paper is organized as follows: In Section 2, a nonlinear load-capacity model is introduced. Network evolution and controllability evolution under cascading failures on BA scale-free networks are numerically simulated and analyzed in Section 3, respectively. Finally, the conclusions are drawn.

2. A NONLINEAR LOAD-CAPACITY MODEL

In load-capacity models, a fundamental assumption is that at each time step, one unit of information is transmitted along the shortest path between each pair of nodes. The edge load is defined as edge betweenness centrality, i.e., the load L_{ij} of edge e_{ij} is the amount of shortest paths going through edge e_{ij} in network. The capacity of an edge is the largest load that the edge can bear, and the capacity H_{ij} of edge e_{ij} is assigned according to its initial load. In the ML model, capacity H_{ij} of edge e_{ij} is linearly proportional to its initial load L_{ij}, i.e.,

$$H_{ij} = (1 + \beta)L_{ij}, \tag{2}$$

where $\beta > 0$ is the tolerance parameter which denotes the portion of additional capacity of edge e_{ij}.

It is found that the removing of the highest-load edges always triggers large scale cascading failures (Motter (2004), Wu *et al.* (2006), Cohen *et al.* (2001)), which change the topology of the networks, therefore, there is a chance that the varying of unoccupied portion of nearly highest-load edges affects the robustness of network controllability. Thus, we introduce a nonlinear load-capacity model (NL model):

$$H_{ij} = L_{ij} + \beta \left(1 - \delta \frac{L_{ij}}{L_{max}}\right) L_{ij}^{\alpha}, \tag{3}$$

where $0 < \alpha \leq 1$ and $0 \leq \delta \leq 1$ decide the unoccupied portion of capacities of high-load edges and nearly highest-load edge, respectively, $\beta \geq 0$ denotes the unoccupied portion of additional capacities of all edges, and $L_{max} = \max(L_{ij})$. When $\delta = 0$, the NL model degenerates to the model proposed by Dou *et al.* (2010). When $\alpha = 1$ and $\delta = 0$, the NL model degenerates to the ML model. The load-capacity relationships of the ML model and the NL model are shown in Fig. 1, respectively. For the ML model, the capacity is linearly proportional to the load (the blue line is parallel to the dotted line), and the NL model has a lager unoccupied portion of capacities on network elements with smaller loads, which is closer to the load-capacity relationship in real complex networks. Parameter α defines the global speed of the capacity converging to the load, and parameter δ determines the capacities of nearly highest-load edges without affecting the global convergence speed. It is obvious that the larger δ, the faster the capacity converging to the load. The effect of adjusting unoccupied portion of capacities of high-load edges and nearly highest-load edges to the controllability will be discussed in the section below by adjusting parameters α and δ, respectively.

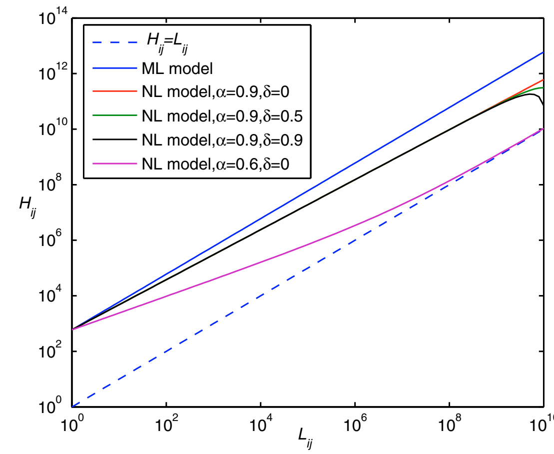

Fig. 1. Two load-capacity models. The dotted line represents the function $H_{ij} = L_{ij}$, and parameter $\beta = 600$ for the ML and the NL models.

For a network, each edge is assigned an initial load, and its capacity is calculated by Equation (3). The edge capacity is limited since it is restricted by the cost of networks, where network cost e is defined as (Dou *et al.* (2010)):

$$e = \sum_{i,j=1}^{N} H_{ij} \Bigg/ \sum_{i,j=1}^{N} L_{ij}. \tag{4}$$

When one edge in the network is removed (especially the one with the highest-load), the distribution of shortest paths changes and the capacities of some edges may become smaller than their loads. A new distribution of edges loads arise after the removing of overloaded edges, and the cascading failure stops until there is no overloaded edge. It is found that the cascading failure of complex networks can easily be triggered by the removing of the highest-load edge (Crucitti *et al.* (2004)). Hence, we consider the cascading failure caused by the removing of the highest-load edge in this paper.