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INCLUSION, RESTRICTION, AND OVERLAPPING DECOMPOSITIONS OF NEUTRAL SYSTEMS WITH DISTRIBUTED TIME-DELAY[1]

Altuğ İftar

Department of Electrical and Electronics Engineering, Anadolu University, 26470 Eskişehir, Turkey. aiftar@anadolu.edu.tr

Abstract: Inclusion and restriction principles for linear time-invariant neutral distributed-time-delay systems is defined. Overlapping decompositions and expansions of such systems are then discussed. Controller design using overlapping decompositions is also presented.

Keywords: Large-scale systems; time-delay systems; neutral systems; distributed time-delay; overlapping decompositions; controller design.

1. INTRODUCTION

Large-scale systems usually involve time-delays. Furthermore, the time-delays in such a system may be distributed (Niculescu (2001)). Such systems can in general be described by functional vector differential equations involving a state vector (Hale and Verduyn-Lunel (1993)). When the derivative of the state vector is not subject to any time-delays, such systems are called as *retarded* time-delay systems. There are, however, examples of systems when the derivative of the state vector may also be subject to time-delays. In the latter case, such systems are called as *neutral* time-delay systems. It is, in general, more difficult to analyze and control neutral systems compared to retarded systems (Niculescu (2001)).

Decomposition techniques are usually needed to analyze large-scale systems or to design controllers for such systems. Many large-scale systems, such as interconnected power systems (Šiljak (1978)), freeway traffic regulation systems (Isaksen and Payne (1973)), intelligent vehicle-highway systems (Stanković et al. (2000)), large flexible structures (Özgüner et al. (1988)), data-communication networks (Ataşlar and İftar (1999)), and manufacturing systems (Aybar and İftar (2002)), however, may involve subsystems which are loosely interconnected among themselves, but strongly interconnected through certain dynamics. The approach of *overlapping decompositions* has first been introduced by Ikeda and Šiljak (1980) to obtain useful decompositions for such systems. The overlapping decompositions is based on the *inclusion principle* (Ikeda et al. (1984)). Although the idea of overlapping decompositions go back to more than three decades, consideration of it for time-delay systems has rather been recent (e.g., Bakule et al. (2005a,b); Bakule and Rossell (2008); İftar (2008)). Furthermore, most of this literature was restricted to systems with discrete time-delays. To the author's best knowledge, systems with distributed time-delay have first been considered by İftar (2014), where the inclusion principle for linear time-invariant (LTI) distributed-time-delay systems has been defined and overlapping decomposition of such systems have been considered. Controller and observer design using overlapping decompositions were also discussed by İftar (2014). The results of İftar (2014), however, were restricted to retarded time-delay systems. Therefore, in this work, we extend the results of İftar (2014) to neutral LTI distributed-time-delay systems. The inclusion principle for such systems is defined in Section 2. An important special case of inclusion, namely restriction, is defined in Section 3. Overlapping decompositions and expansions and controller design are discussed in Section 4. Finally, some concluding remarks are given in Section 5.

Throughout the paper, for positive integers k and l, $\mathbf{R}^k$ and $\mathbf{R}^{k\times l}$ denote the spaces of, respectively, k-dimensional real vectors and $k\times l$-dimensional real matrices. I_k denotes the $k\times k$-dimensional identity matrix. 0 may denote either the scalar zero, a zero vector, a zero matrix, or a matrix function which is identically zero. For real numbers a and b, $[a,b] := \{\rho \mid a \le \rho \le b\}$ and $[a,b) := \{\rho \mid a \le \rho < b\}$. For a complex number s, $\mathrm{Re}(s)$ is the real part of s. Finally, for a vector function $x(\cdot)$, $\dot{x}(\cdot)$ is the derivative of $x(\cdot)$.

2. INCLUSION

In this section, we extend the inclusion principle to LTI neutral systems with distributed time-delay, which can be described as:

$$\dot{x}(t) + \int_{-\tau}^{0} E(\theta)\dot{x}(t+\theta)d\theta = \int_{-\tau}^{0} \left(A(\theta)x(t+\theta) + B(\theta)u(t+\theta)\right)d\theta \qquad (1)$$

$$y(t) = \int_{-\tau}^{0} C(\theta)x(t+\theta)d\theta$$

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where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^p$, and $y(t) \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors at time t, τ is the maximum time-delay in the system, and $E(\cdot) : [-\tau, 0] \to \mathbf{R}^{n\times n}$, $A(\cdot) : [-\tau, 0] \to \mathbf{R}^{n\times n}$, $B(\cdot) : [-\tau, 0] \to \mathbf{R}^{n\times p}$, and $C(\cdot) : [-\tau, 0] \to \mathbf{R}^{q\times n}$ are bounded matrix functions, except that they may involve Dirac delta terms. The inclusion of Dirac delta terms in those matrices allow the representation of discrete time-delays besides distributed time-delays. $E(0)$, however, is assumed to be bounded, i.e., $E(\theta)$ can have Dirac delta terms $\delta(\theta - h)$, for $h \in [-\tau, 0)$, but not for $h = 0$. We will denote the system described by (1) by Σ, and also consider another LTI neutral system with distributed time-delay, to be denoted by $\hat{\Sigma}$, described as:

$$\dot{\hat{x}}(t) + \int_{-\tau}^{0} \hat{E}(\theta)\dot{\hat{x}}(t+\theta)d\theta = \int_{-\tau}^{0} \left(\hat{A}(\theta)\hat{x}(t+\theta) + \hat{B}(\theta)\hat{u}(t+\theta)\right) d\theta \qquad (2)$$

$$\hat{y}(t) = \int_{-\tau}^{0} \hat{C}(\theta)\hat{x}(t+\theta)d\theta$$

where, $\hat{x}(t) \in \mathbf{R}^{\hat{n}}$, $\hat{u}(t) \in \mathbf{R}^p$, and $\hat{y}(t) \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors at time t and $\hat{E}(\cdot) : [-\tau, 0] \to \mathbf{R}^{\hat{n}\times\hat{n}}$, $\hat{A}(\cdot) : [-\tau, 0] \to \mathbf{R}^{\hat{n}\times\hat{n}}$, $\hat{B}(\cdot) : [-\tau, 0] \to \mathbf{R}^{\hat{n}\times p}$, and $\hat{C}(\cdot) : [-\tau, 0] \to \mathbf{R}^{q\times\hat{n}}$ are bounded matrix functions, except that they may involve Dirac delta terms. As for $E(0)$, however, $\hat{E}(0)$ is assumed to be bounded. We note that, although both Σ and $\hat{\Sigma}$ are assumed to have the same maximum time-delay, τ, this is no loss of generality, since, if one system has a longer maximum time-delay, the matrix functions of the other system can be extended as zero matrices down to the common maximum time-delay. Furthermore, it is assumed that the input, as well as output, vectors of Σ and of $\hat{\Sigma}$ have the same dimensions (p and q respectively). However, the state vector of $\hat{\Sigma}$ has a larger dimension than that of Σ; i.e., $\hat{n} > n$. Finally, the initial conditions for Σ and $\hat{\Sigma}$ are assumed to be respectively given as:

$$x(\theta) = \phi(\theta) \quad \text{and} \quad \hat{x}(\theta) = \hat{\phi}(\theta) , \quad \theta \in [-\tau, 0] , \quad (3)$$

for some functions $\phi : [-\tau, 0] \to \mathbf{R}^n$ and $\hat{\phi} : [-\tau, 0] \to \mathbf{R}^{\hat{n}}$.

Unlike retarded time-delay systems (which were discussed in İftar (2014)), neutral time-delay systems may have infinitely many modes in a given right-half plane (Niculescu (2001)). It is, however, known that (Hale and Verduyn-Lunel (1993)), for any $\rho > 0$, the system Σ has only finitely many modes with real part greater than or equal to $\mu(\Sigma) + \rho$, where

$$\mu(\Sigma) := \sup\left\{ \mathrm{Re}(s) \mid \det\left(I_n + \int_{-\tau}^{0} E(\theta)e^{s\theta}d\theta \right) = 0 \right\} . (4)$$

Similarly, for any $\rho > 0$, the system $\hat{\Sigma}$ has only finitely many modes with real part greater than or equal to $\mu(\hat{\Sigma}) + \rho$, where $\mu(\hat{\Sigma})$ is defined as in (4) with I_n and $E(\theta)$ respectively replaced by $I_{\hat{n}}$ and $\hat{E}(\theta)$. It is known that a system of the form Σ can not be stabilized by a proper controller unless $\mu(\Sigma) < 0$ (Loiseau et al. (2002)). Therefore, in the sequel, we assume that $\mu(\Sigma) < 0$ and $\mu(\hat{\Sigma}) < 0$.

Now, we present the following definition, which is an extension of the definition in İftar (2014) to the case of neutral systems.

Definition 1: $\hat{\Sigma}$ *includes* Σ and Σ is *included by* $\hat{\Sigma}$ if there exist a full row-rank matrix $U \in \mathbf{R}^{n\times\hat{n}}$ and a full column-rank matrix $V \in \mathbf{R}^{\hat{n}\times n}$ with $UV = I_n$, such that for all $\phi(\cdot)$ and for all $u(\cdot)$, the choice

$$\hat{\phi}(\theta) = V\phi(\theta) , \quad \theta \in [-\tau, 0] \qquad (5)$$

and

$$\hat{u}(t) = u(t) , \quad t \geq -\tau \qquad (6)$$

implies

$$x(t) = U\hat{x}(t) , \quad t \geq -\tau \qquad (7)$$

and

$$y(t) = \hat{y}(t) , \quad t \geq 0 . \qquad (8)$$

When $\hat{\Sigma}$ includes Σ, the two systems have the same input-output relation and some stability properties are preserved. First, let us present the following definitions, which are borrowed from İftar (2014):

Definition 2: Two systems with the same number of inputs and outputs, such as Σ and $\hat{\Sigma}$, are said to *have the same input-output map* if, for any input, they produce the same output in response to the same input when their initial conditions are zero.

Definition 3: A system, such as Σ or $\hat{\Sigma}$, is said to be *bounded-input bounded-output (BIBO) stable* if, in response to any bounded input, it produces a bounded output when its initial condition is zero.

Definition 4: A system, such as Σ or $\hat{\Sigma}$, is said to be *(asymptotically) stable* if, for any bounded initial condition, its state remains bounded (and asymptotically goes to zero as time goes to infinity) when its input is zero.

The following theorems show that, when $\hat{\Sigma}$ includes Σ, the two systems have the same input-output map and certain stability properties are preserved between them.

Theorem 1: If $\hat{\Sigma}$ includes Σ, then Σ and $\hat{\Sigma}$ have the same input-output map.

Proof: When Σ and $\hat{\Sigma}$ have both zero intial conditions, i.e., when $\phi(\theta) = 0$ and $\hat{\phi}(\theta) = 0$, $\forall\theta \in [-\tau, 0]$, then (5) is satisfied. Furthermore, when the two systems have the same input, then (6) is satisfied. Then, since $\hat{\Sigma}$ includes Σ, (8) is also satisfied, which is the desired result. □

Theorem 2: If $\hat{\Sigma}$ includes Σ, then Σ is BIBO stable if and only if $\hat{\Sigma}$ is BIBO stable.

Proof: Follows from Definitions 2 & 3 and Theorem 1. □