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Model Predictive Control for Collision Avoidance of Networked Vehicles Using Lagrangian Relaxation ⋆

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Abstract: This paper focuses on designing a steering intervention system based on Model Predictive Control. The goal is to avoid collisions between networked vehicles. Since the resulting program is non-convex, it is converted to a convex Semi-Definite Program by using Lagrangian relaxation. Furthermore, the utilized prediction vehicle model is linearized in an exact way to avoid errors due to linear approximation. Initially, the whole problem is formulated. Finally, the performance of the controller is presented in Model-in-the-Loop simulations.

Keywords: Model Predictive Control, Non-convex Optimization, Lagrangian Relaxation, Collision Avoidance, Networked Vehicles.

1. INTRODUCTION

The proposed algorithms in this paper are developed for collision avoidance of road vehicles. Nonetheless, they can be applied in other fields such as networked aerial, water and ground vehicles. In terms of road traffic, the World Health Organization published its status report on road safety in 2013. It is said that 1.24 million people die on the world's roads every yearWHO (2013). Although the mentioned number has not significantly changed since 2007, it is an important issue to find new solutions which lead to a decrease of this number, since road traffic injuries are still the eighth leading cause of death all over the world. This paper focuses on the design of a steering intervention system based on Model Predictive Control (MPC). Due to the recent developments in vehicle communication, a controller can be implemented for collision avoidance of networked vehicles. This controller should be able to solve the current optimization problem at each time step. There are two main tasks. The first task is to avoid any collision with opponent vehicles and obstacles. As a second task, the controller should keep each vehicle as close as possible to its reference trajectory (desired trajectory). Due to these partly contradictory requirements, each controlled vehicle will drive a collision-free trajectory which only deviates from the reference trajectory in case of imminent collisions. In the setup, it is distinguished between active members (vehicles) and passive ones (obstacles). Obstacles are either stationary or moving along a known path. The changes in the vehicle's steering angle with respect to the changes in the time step are penalized within the optimization problem's objective function to keep them smooth. Furthermore, a maximum absolute steering angle is defined in the input constraints to limit the lateral acceleration.

First, in section 2, the linearized and time-discretized prediction vehicle model and the optimization problem are presented. Since the vehicles and obstacles are described as circles or ellipses, the resulting collision avoidance constraints make the optimization problem a non-convex Quadratically Constrained Quadratic Program (QCQP). However, it can be converted into a convex Semi-Definite Program (SDP) through Lagrangian relaxation, which is described in section 3. An exact linearization of the vehicle model is also introduced to avoid errors due to linear approximation. Section 4 shows the simulation results which demonstrate the controller's performance. Finally, in section 5, a conclusion is given.

2. SETUP OF THE COLLISION AVOIDANCE PROBLEM

In this section, the basic problem for the MPC is formulated. The main purpose of the controller is to find appropriate steering angles for each time step such that each vehicle follows the given reference trajectory while preventing collisions at the same time. In case of an imminent collision, the reference trajectory has to be left in order to avoid the collision. After collision avoidance, the vehicle should return to the reference trajectory. In this context, a vehicle model is needed to predict the future behavior of the system. At each discrete time step, the current optimization problem has to be solved Maciejowski (2002).

The MPC has access to all relevant information about the vehicles and obstacles, their states, and their reference trajectories. The controller should solve an optimal control problem by considering the benefits of all vehicles and the defined constraints. The set of constraints considers the vehicle collision avoidance between each two vehicles, obstacle collision avoidance between each vehicle and obstacle, and the constraints on the control input. The solution of the MPC is system-wide optimal since

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the whole optimization problem is solved at once and the system feasibility collision avoidance is guaranteed through Vehicle-to-Vehicle (V2V) communication.

2.1 Vehicle Dynamics

The chosen vehicle model, which is used for the state prediction over the prediction horizon, is a nonlinear kinematic bicycle model, as introduced in Rajamani (2005), (see Fig. 1). R defines the radius of the driven circle around the center point O. For simplification, the following

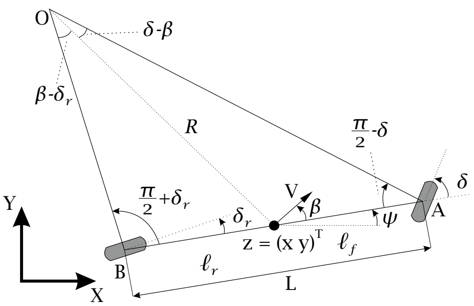

Fig. 1. Kinematic bicycle model of the vehicle

assumptions are made: (i) the height of the center of gravity (COG) is zero, hence pitch and roll dynamics are neglected, (ii) the front wheels are represented by one wheel at point A, and the rear wheels at B, (iii) rolling resistances and aerodynamic drag are neglected, (iv) the vehicle is front-wheel-only steering ($\delta_r = 0$), (v) the slip angle β between the velocity vector and longitudinal direction of the vehicle is neglected and (vi) no forces are applied neither at the front nor at the rear tire. The resulting state space system can be formulated as follows:

$$\begin{aligned} &\dot{x} = v\cos(\psi), \quad \dot{y} = v\sin(\psi), \quad \dot{\psi} = \frac{v}{L}\tan\delta, \\ &\dot{v} = a, \qquad\qquad \dot{a} = 0, \end{aligned} \tag{1}$$

where $z = (x\ y)^T$ indicates the location of the COG, ψ the yaw angle with respect to the X-axis, $L = l_r + l_f$ the wheelbase, v the velocity, and a the acceleration. Additionally it is assumed that $a = 0$. Therefore, $\dot{a} = 0$ is not needed. But it is introduced due to the exact linearization, which will be described in section 3.2.1. Here, $(x\ y\ \psi\ v\ a)^T$ is the state vector, z is the controlled output, and the steering angle δ is the control input. In the MPC algorithm, the nonlinear prediction model (1) is linearized using Taylor series around a particular operating point $(x_0, y_0, \psi_0, v_0, a_0, \delta_0)$ Khalil (2002). Katriniok and Abel (2011) shows that an improved control performance can be achieved using successive linearization compared to unchanged linearization. Additionally, the model is time-discretized. The resulting prediction model can be written as follows:

$$\begin{aligned} X_i(t+1) &= A_i(t)X_i(t) + B_i(t)u_i(t) + E_i(t) \\ z_i(t) &= \underbrace{\begin{pmatrix} 1\ 0\ 0\ 0\ 0 \\ 0\ 1\ 0\ 0\ 0 \end{pmatrix}}_{C} X_i(t), \end{aligned} \tag{2}$$

where $i = 1, \ldots, N_{veh}$ denotes the vehicle's number, $X_i \in \mathbb{R}^5$, and $u_i \in \mathbb{R}$. The system matrix $A_i(t) \in \mathbb{R}^{5\times5}$, the input matrix $B_i(t) \in \mathbb{R}^5$ and the affine term $E_i(t) \in \mathbb{R}^5$, that results from the linearization of (1), are time-variant matrices. Moreover, it is $u_i \in \mathcal{U}$, where $\mathcal{U}$ indicates the set of allowed control inputs. $\mathcal{U}$ is limited in order not to exceed a maximum lateral acceleration which is defined as $a_{y,max} = 0.5g$, where $g \approx 10\frac{m}{s^2}$ is the acceleration due to gravity. According to Rajamani (2005), $\delta_{max} \approx tan^{-1}(\frac{a_{y,max}L}{v^2})$. Due to computational time issues, the kinematic model is used instead of a complex one that considers lateral forces.

2.2 Obstacle Collision Avoidance

The obstacles are assumed to be fixed or moving with a known direction and velocity. Thus, their positions and directions in the prediction can be determined exactly. Let $p_o(t+k)$ describe the position of obstacle o at time $t+k$. Assuming that the COGs are equal to the center positions of the vehicles and obstacles, the obstacle collision avoidance constraints for vehicle i can be expressed as follows:

$$\begin{aligned} \|z_i(t+k) - p_o(t+k)\|_2^{S_{e,i,o}} \geq 1, \ & o = 1, \ldots, N_o, \\ & k = 1, \ldots, H_p, \end{aligned} \tag{3}$$

where $z_i(t+k)$ denotes the predicted position of vehicle i at time $t+k$ starting from time t. N_o is the number of obstacles and H_p is the prediction horizon. Furthermore,

$$S_{e,i,o} = rot_o^T \begin{pmatrix} \frac{1}{\alpha_{x,i,o}^2} & 0 \\ 0 & \frac{1}{\alpha_{y,i,o}^2} \end{pmatrix} rot_o, \tag{4}$$

with the rotation matrix

$$rot_o = \begin{pmatrix} cos(\psi_o) & sin(\psi_o) \\ -sin(\psi_o) & cos(\psi_o) \end{pmatrix}, \tag{5}$$

specifies the ellipse around the center position of obstacle o which should not be entered by the center point of vehicle i in order to avoid a collision. The ellipse is rotated with the yaw angle ψ_o of obstacle o. The variables $\alpha_{x,i,o}$ and $\alpha_{y,i,o}$ are the ellipse constants which can also define the radius of a circle in case of $\alpha_{x,i,o} = \alpha_{y,i,o}$. Since the dimensions of each vehicle $i = 1, \ldots, N_{veh}$ have to be considered for the determination of the constants, each obstacle is described by up to N_{veh} different ellipses.

2.3 Vehicle Collision Avoidance

Based on the assumptions for the obstacle collision avoidance constraints, the collision avoidance constraints for the vehicles i and j can be formulated as follows:

$$\|z_i(t+k) - z_j(t+k)\|_2^{S_{c,i,j}} \geq 1, \ j > i, \ k = 1, \ldots, H_p, \tag{6}$$

where

$$S_{c,i,j} = \begin{pmatrix} \frac{1}{\alpha_{i,j}^2} & 0 \\ 0 & \frac{1}{\alpha_{i,j}^2} \end{pmatrix}, \tag{7}$$

specifies the circle with the radius $\alpha_{i,j}$ around the center position of vehicle j which should not be entered by the center point of vehicle i in order to avoid a collision. Because of the rotary symmetry of circles, $\alpha_{i,j} = \alpha_{j,i}$. Since the vehicles change their yaw angles over the prediction horizon, the trigonometric terms of their rotated ellipses are not constant. In this case, it is not possible to determine quadratic constraints for a QCQP. Hence, the vehicles are only described as circles.