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Elucidation of the role of constraints in economic model predictive control

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ABSTRACT

Economic model predictive control (EMPC) is a predictive feedback control methodology that unifies economic optimization and control. EMPC uses a stage cost that reflects the process/system economics. In general, the stage cost used is not a quadratic stage cost like that typically used in standard tracking model predictive control. In this paper, a brief overview of EMPC methods is provided. In particular, the role of constraints imposed in the optimization problem of EMPC for feasibility, closed-loop stability, and closed-loop performance is explained. Three main types of constraints are considered including terminal equality constraints, terminal region constraints, and constraints designed via Lyapunov-based techniques. The paper closes with a well-known chemical engineering example (a non-isothermal CSTR with a second-order reaction) to illustrate the effectiveness of time-varying operation to improve closed-loop economic performance compared to steady-state operation and to demonstrate the impact of economically motivated constraints on optimal operation.

1. Introduction

Economic model predictive control (EMPC) has attracted significant attention and research over the last five years. This interest is a result of the ability of EMPC to integrate optimization of process economics with process control by incorporating a general stage cost function in the optimization problem and allowing for consistently dynamic (time-varying) process operation without requiring the process to settle at a steady-state or reference trajectory (Amrit, Rawlings, and Angeli, 2011; Angeli, Amrit, and Rawlings, 2012; Engell, 2007; Heidarinejad, Liu, and Christofides, 2012a; Helbig, Abel, and Marquardt, 2000; Huang, Harinath, and Biegler, 2011; Rawlings and Amrit, 2009; see, also, the reviews Ellis, Durand, and Christofides, 2014; Rawlings, Angeli, and Bates, 2012 for a more complete overview and reference list of the EMPC literature). In contrast to tracking model predictive control (MPC), which usually incorporates a quadratic stage cost, the stage cost of EMPC is chosen as a direct or indirect measure of the process/system economic performance. As a result of the general stage cost used, EMPC may force a process to operate in a time-varying manner to optimize the economics. The rigorous design of EMPC schemes that operate large-scale processes in a dynamically optimal fashion while maintaining stability of the closed-loop system is challenging because traditional stability analysis concepts, such as asymptotic stability of a steady-state for a process under a given controller, may be inapplicable to a closed-loop system under EMPC.

To address the three key fundamental issues of feasibility, stability, and economic performance, constraints are often employed in the EMPC problem formulation. To this end, many EMPC formulations have been proposed encompassing theoretical analysis of closed-loop properties (e.g., Alessandretti, Aguiar, & Jones, 2014; Amrit et al., 2011; Angeli et al., 2012; Bayer, Müller, & Allgöwer, 2014; Faulwasser, Korda, Jones, & Bonvin, 2014; Ferramosca, Rawlings, Limon, & Camacho, 2010; Grüne, 2013; Grüne & Stieler, 2014; Heidarinejad et al., 2012a; Huang, Biegler, & Harinath, 2012; Huang et al., 2011; Limon, Pereira, Muñoz de la Peña, Alamo, & Grosso, 2014; Müller, Angeli, & Allgöwer, 2014a; Zavala, 2015), optimization and computational issues (e.g., Biegler, Yang, & Fischer, 2015; Kadam & Marquardt, 2007; Würth & Marquardt, 2014), and implementation and applications (e.g., Ellis & Christofides, 2015b; Grosso, Ocampo-Martinez, Puig, Limon, & Pereira, 2014; Heidarinejad, Liu, & Christofides, 2012b; Omell & Chmielewski, 2013; Zhang, Liu, & Liu, 2014).

This article describes the role and implications of constraints used in EMPC. It is an extended version of the work (Ellis & Christofides, 2015a). Owing to space limitations, certain technical assumptions are omitted and statements of the results are summarized.

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Notation: $|\cdot|$ denotes the Euclidean norm of a vector. The symbol $S(\Delta)$ denotes the family of piecewise constant functions with period $\Delta > 0$. A continuous function $\beta : \mathbb{R}^n \to \mathbb{R}$ is positive definite if $\beta(0) = 0$ and $\beta(x) > 0$ for all $x \neq 0$. A continuous function α: $[0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it strictly increasing and $\alpha(0) = 0$.

1.1. Class of nonlinear systems

The class of systems considered is described by the system of nonlinear ordinary differential equations (ODEs):

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

where $x(t) \in \mathbb{X} \subset \mathbb{R}^n$ denotes the state vector, $u(t) \in \mathbb{U} \subset \mathbb{R}^m$ denotes the manipulated (control) input vector, and $w(t) \in \mathbb{W} \subset \mathbb{R}^l$ denotes the disturbance vector. The set of admissible input values $\mathbb{U}$ is compact, and the disturbance vector is bounded in the set $\mathbb{W} := \{w \in \mathbb{R}^l \,|\, |w| \leq \theta\}$ where $\theta > 0$ bounds the norm of the disturbance vector. The vector function $f : \mathbb{X} \times \mathbb{U} \times \mathbb{W} \to \mathbb{X}$ is locally Lipschitz on $\mathbb{X} \times \mathbb{U} \times \mathbb{W}$. A state measurement is synchronously sampled at sampling instances denoted as $t_k := k\Delta$ where $k \in \mathbb{I}_{\geq 0}$ and $\Delta > 0$ is the sampling period (the initial time is taken to be zero). The assumption of state feedback is standard owing to the fact that the separation principle does not generally hold for nonlinear systems. Nevertheless, some rigorous output feedback implementations of EMPC exist (e.g., Heidarinejad et al., 2012b; Zhang et al., 2014). The system (1) is equipped with a continuous function $l_e : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$, which reflects the instantaneous process/system economics. The function $l_e(\cdot, \cdot)$ will be used as a stage cost in a model predictive control (MPC) framework and will be referred to as the economic stage cost. The system (1) may have additional constraints other than the input and state constraints. Collecting all the constraints including the input, state, and additional constraints, the constraints may be written generally as static constraints:

$$g_s(x, u) \leq 0 \quad (2)$$

where $g_s : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^{n_s}$ and as dynamic constraints (e.g., average constraints):

$$\int_0^{t_d} g_d(x(t), u(t))dt \leq 0 \quad (3)$$

where $g_d : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^{n_d}$ and t_d is the time horizon that the constraint is imposed. The dynamic constraints are often motivated by economic considerations. The economically optimal steady-state and steady-state input pair is:

$$(x_s^*, u_s^*) = \underset{(x_s, u_s)}{\arg\min} \left\{ l_e(x_s, u_s) : \begin{array}{c} f(x_s, u_s, 0) = 0 \\ g_s(x_s, u_s) \leq 0, \\ g_d(x_s, u_s) \leq 0 \end{array} \right\}. \quad (4)$$

With the notation above, the optimal steady-state pair (x_s^*, u_s^*) is assumed to be unique. If the minimizing pair is not unique, let (x_s^*, u_s^*) denote one of the minimizing steady-state pairs. The optimal steady-state is taken to be the origin of the unforced system ($f(0, 0, 0) = 0$).

Remark 1. Time-varying economic considerations such as customer demand changes, dynamic energy pricing, and variable feedstock quality may lead to explicitly time-varying economic stage costs as well as time-dependent economic-oriented constraints. While economic stage costs that are not explicitly time-dependent are considered here, some EMPC methodologies exist for handling some issues related to time-varying economic stage costs such as a Lyapunov-based EMPC formulation that allows for changing regions of operation as the economic stage cost changes with time while guaranteeing closed-loop stability (Ellis & Christofides, 2014a). Another potentially useful concept that may help enable EMPC to handle time-varying economic stage costs is the use of a generalized terminal constraint or self-tuning terminal region and terminal cost (e.g., Fagiano & Teel, 2013; Ferramosca et al., 2010; Müller et al., 2014a).

2. EMPC schemes: feasibility, closed-loop stability, and performance

Economic model predictive control is an MPC method that uses the economic stage cost in its formulation. The EMPC problem, with a finite-time prediction horizon, can be broadly characterized by the following optimal control problem (OCP):

$$\min_{u(\cdot) \in S(\Delta)} \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(t), u(t))dt + V_f(\tilde{x}(t_{k+N})) \quad (5a)$$

$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \quad (5b)$$

$$\tilde{x}(t_k) = x(t_k) \quad (5c)$$

$$g_s(\tilde{x}(t), u(t)) \leq 0, \forall t \in [t_k, t_{k+N}] \quad (5d)$$

$$\int_{t_k}^{t_{k+N}} g_d(\tilde{x}(t), u(t))dt \leq 0 \quad (5e)$$

where the decision variable of the optimization problem is the piecewise constant input trajectory over the prediction horizon (i.e., the time interval $[t_k, t_{k+N})$) and $\tilde{x}$ denotes the predicted state trajectory over the prediction horizon. Higher order control parameterizations may also be considered. Nevertheless, sample-and-hold (i.e., zeroth-order hold) implementation of controls is one of the most commonly employed control parameterizations (i.e., $u(\cdot) \in S(\Delta)$ as in (5a)).

The cost functional (5a) consists of the economic stage cost with a terminal cost/penalty $V_f : \mathbb{X} \to \mathbb{R}$. The nominal dynamic model (5b) is used to predict the future evolution of the system and is initialized with a state measurement (5c). When available, disturbance estimates or predictions may be incorporated in the model (5b). The constraints (5d) and (5e) represent the system constraints which may include input, state, mixed state and input, economic, and stability constraints. The constraint (5e) may be time-varying (i.e., formulated for the sampling time t_k, so that the constraint (3) is satisfied over the desired operating interval). With slight abuse of notation, (5e) is not necessarily the same as (3). For the remainder of this section, the dynamic constraints are dropped and only EMPC schemes of the form (5a)–(5d) are considered, except for a brief discussion of the impact of dynamic constraints on the trajectories of EMPC with input rate of change constraints. Thus, the constraint set is $\mathbb{Z} := \{(x, u) : x \in \mathbb{X}, u \in \mathbb{U}, g_s(x, u) \leq 0\} \subseteq \mathbb{X} \times \mathbb{U}$ and $\mathbb{Z}$ is assumed to be compact.

Like tracking MPC, EMPC is typically implemented with a receding horizon implementation to better approximate the infinite horizon solution and to ensure robustness of the control solution to disturbances and open-loop instabilities. At a sampling time t_k, the EMPC receives a state measurement, which is used to initialize the model (5b). The OCP (5) is solved on-line for a (local) optimal piecewise input trajectory, denoted by $u^*(t|t_k)$ for $t \in [t_k, t_{k+N})$. The control action computed for the first sampling period of the prediction horizon, denoted as $u^*(t_k|t_k)$, is sent to the control actuators to be implemented over the sampling period from t_k to t_{k+1} (i.e., sample-and-hold implementation). At the next sampling time, the OCP (5) is re-solved after receiving a new state measurement and by shifting the prediction horizon into the future by one sampling period.

EMPC, which consists of the on-line solution of the OCP (5) along with a receding horizon implementation, results in an implicit state feedback law $u(t) = \kappa(x(t_k))$ for $t \in [t_k, t_{k+1})$. From