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# Modeling populations of electric hot water tanks with Fokker-Planck equations

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**Abstract:** This article proposes a distributed parameters model for a pool of electric hot water tanks (EHWT). EHWT are electric appliances found in numerous homes where they produce hot water for domestic usages. Designing smart piloting for them requires a careful description of several variables of interest and their dynamics. When the number of such devices is large, these dynamics can be lumped into Fokker-Planck equations. In this case, these equations are driven by in-domain control which defines the heating policies in a stochastic manner. The main contribution of this article is the Fokker-Planck model of a pool of EHWT.



*Keywords:* Water heating, Electric Hot Water Tank, Smart piloting, Fokker-Planck equation

## 1. INTRODUCTION

The increasing share of intermittent renewable electricity sources in the energy mix (European Commission [2011], Edenhofer et al. [2011]) raises new difficulties in management of the electricity production and equilibrium in distribution networks. Demand Side Management (DSM), which is a portfolio of smart piloting techniques aiming at modifying consumers' demand, is a promising solution for such concerns (Palensky and Dietrich [2011]). A key factor in developing DSM is the ability to find energy storage capacities. In this context, the large pools of electric hot water tanks (EHWT) have a well recognized potential.

An EHWT is a domestic electric appliance which heats a volume of water with an electric thermo-plunger that can be controlled. The home user drains hot water from the EHWT at various times of the day. The literature (Blandin [2010], Zurigat et al. [1991] and the references therein) models EHWT as vertical cylindrical tanks driven by thermo-hydraulic phenomena: heat diffusion, buoyancy effects and induced convection and mixing, forced convection induced by draining and associated mixing, and heat loss at the walls.

To model EHWT, one-dimensional distributed parameter models of the temperature profile in the tank have been developed (see Beeker et al. [2015a,b]). The observed temperature profiles are increasing with height. For smart piloting applications, one can define three variables of interest providing a simplified representation of the temperature profile, under the form of three amounts of energy. From this, advanced control designs can be studied. Among them, optimal control strategies are particularly appealing as they address topics of direct interest both for end-users and electricity producers: cost reduction, comfort constraints, and yield management. Yet, the situation is not that straightforward. In truth, a real challenge lies in solving optimal control problems for large numbers of such EHWT. Coordinated individual control of each tank is feasible for pools of moderate sizes (typically from 2 to 10 EHWT). However, for large pools of EHWT (tens of thousands), this approach is a stalemate. Unfortunately, the real stakes and industrial expectations belong to this range.

Interestingly, it is possible to recast this problem into a distributed parameters approach. This is the path we explore in this article. Following the works of Malhamé and Chong [1985], we consider that the local (individual) control variables of EHWT are each defined according to stochastic processes. Then, we combine *i)* this randomness, *ii)* the diversity in the distribution of the states of the EHWT, *iii)* the randomness of the water consumptions, and we develop a partial differential equation (PDE) for a large pool of EHWT. This takes the form of Fokker-Planck equations (see Risken [1996]) governing the probability distributions of the population of EHWT. The work of Malhamé and Chong and the following ones (see e.g. Moura et al. [2013]) were originally focused on a mitigated load represented by a single state, which we need to extend for the smart piloting applications under consideration here. This extension results into a rich system of PDE, which constitutes the main contribution of this paper.

The paper is organized as follows. In the pool, a single EHTW is a macroscopic but small subsystem described by state variables. These variables of interest are defined in Section 2. They have hybrid dynamics by construction. To account for the randomness of water consumption, we propose a single EHWT model as a Markovian stochastic process in Section 3. Then, we introduce probability density function of the population of EHWT and derive the Fokker-Planck equations in Section 4. A summary of

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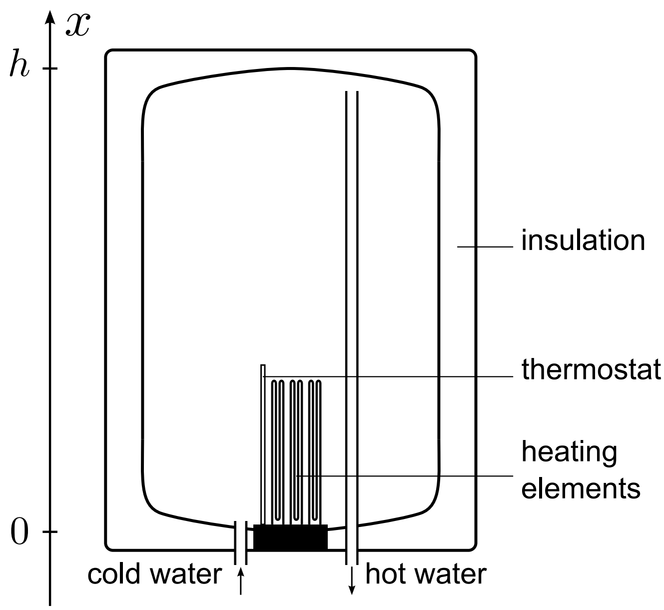


Fig. 1. Schematic view of an electric hot water tank.

the obtained input-output description of the EHWT pool is reported in Section 5. Conclusions and perspectives are given in Section 6.

## 2. VARIABLES OF INTEREST IN A EHWT

### *2.1 General description, stratification, and effects of heating*

A typical EHWT is a vertical cylindrical tank filled with water. A heating element is plunged at the bottom end of the tank (see Fig. 1). The heating element is pole-shaped, and relatively lengthy, up to one third of the tank. Cold water is injected at the bottom while hot water is drained from the top at exactly the same flow-rate (under the assumption of pressure equilibrium in the water distribution system). In the tank, layers of water with various temperature coexist (see Fig.2). At rest, these layers are mixed only by heat diffusion which effects are relatively slow compared to the other phenomena (Han et al. [2009]). Existence of a non uniform (increasing with height) quasi-equilibrium temperature profile in the tank is called *stratification* (Dincer and Rosen [2010], Lavan and Thompson [1977]). In practice, this effect is beneficial for the user as hot water available for consumption is naturally stored near the outlet of the EHWT, while the rest of the tank is heated (see Fig.2(b)). Due to this effect and the cylindrical symmetry of the system, one can assume that the water temperature in the tank is homogeneous at each height.

Following the description above, the temperature $T$ of the tank is a continuously increasing function of height (see Fig. 2). The constant inlet temperature $T_{in}$ constitutes a lower bound of the temperature profile. The heating process is driven by turbulence generated by buoyancy effects, which is the cause of a local mixing in the bottom of the tank. We consider that this mixing is perfect on a spatial zone, refereed to as the *plateau* (see Beeker et al. [2015a,b]), and does not affect the temperature profile in the upper part of the tank (see Fig. 2(b)). During the heating process, the plateau grows and gradually covers the whole tank. The user specifies a temperature $T_{max}$ at which the heating has to be stopped to prevent overheating. As a result of the heating process, if the temperature at the bottom of the tank is $T_{max}$, then the temperature in the tank is uniformly at $T_{max}$ once the heating is finished.

The user can also specify a comfort temperature $T_{com}$. Water having temperature higher than $T_{com}$ can be blended with cold water to reach $T_{com}$ and is therefore useful, while water having temperature lower than $T_{com}$ is useless.

### *2.2 Consumption, control and objectives*

Each EHWT has two inputs: water consumption and heating power. The user consumes certain quantities of energy each day. For this reason, consumption of hot water is an (uncontrolled) input of our problem. On the other hand, the heat injected via the heating element in the tank is a control variable.

The control design can have various objectives. The most obvious one is individual cost reduction for each single unit in response to a price signal. At larger scales, one can naturally consider a pool of tanks, and aim at reaching a load profile for the aggregate consumption.

### *2.3 Variables of interest: Available, delay and reserve energies*

Describing the exact temperature profile inside the tank is unnecessary for the applications discussed above. Instead, a few (3) variables of interest can be considered.

The *available energy* $a$ is defined as the energy contained in the zones having temperature greater than the comfort temperature $T_{com}$. This constitutes a direct comfort index for the user. If $a$ reaches 0 and a water drain is applied, then the comfort constraints is violated.

The *delay energy* $\tau$ is defined as the energy required by the plateau to reach the temperature $T_{com}$. When the tank is heated at constant maximum power, in the absence of drains and heat losses, $\tau$ is simply proportional to the time necessary to reach a state from which $a$ can effectively be positively impacted by the heating process.

The *reserve energy* $\mu$ is defined as the energy contained in the tank that is currently unavailable for consumption, i.e. the energy contained in the water under $T_{com}$. When, thanks to the heating process, $\tau$ reaches the value 0, the energy $\mu$ becomes available for consumption: this generates an immediate (discontinuous) increase of $a$, and $\mu$ is reset to 0. [1]

Fig. 2 illustrates the dynamics of these variables. A drain (pictured in (a)) is mainly characterized by a decrease of $a$ and an increase of $\tau$, with a slight raise of $\mu$ due to an energy transfer from $a$. On the other hand, in the heating pictured in (b), $\tau$ decreases at the same rate as $\mu$ rises, until the plateau reach $T_{com}$. An energy transfer from $\mu$ to $a$ takes place then. This is pictured in (c). A case where comfort constraint is violated is pictured in (d).

[1] The rationale behind these definitions is that to plan the heating, we account for the time left before the energy reserve embodied by $a$ (in the total energy $a + \mu$) is consumed, and the time necessary to provide new hot water, embodied by $\tau$.