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Nonlinear observer design for a first order hyperbolic PDE: application to the estimation of the temperature in parabolic solar collectors ⋆

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Abstract: In this paper, the problem of estimating the distributed profile of the temperature along the tube of a concentrated distributed solar collector from boundary measurements is addressed. A nonlinear observer is proposed based on a nonlinear integral transformation. The objective is to force the estimation error to follow some stable transport dynamics. Convergence conditions are derived in order to determine the observer gain ensuring the stabilization of the estimation error in a finite time. Numerical simulations are given to show the effectiveness of the proposed algorithm under different working conditions.

Keywords: Nonlinear observer, Hyperbolic PDE, Parabolic solar collector.

1. INTRODUCTION

In the last few decades, an increasing interest has been given to renewable energy in order to reduce the dependence on fossil resources and their environmental impact. To meet the exponential growth of energy demand, solar energy presents a promising alternative clean source. In particular, the parabolic distributed technology has been widely used in several plants which are already successfully operating or under construction. Concentrated distributed solar collectors consist of parabolic shaped mirrors which concentrate the received sunlight into a receiver tube placed in the central line of the collectors in order to heat the thermal carrier fluid flowing in it (Camacho et al. (1997); Lemos et al. (2014)).

Due to the intermittent and unpredictible variations of the external disturbances affecting the solar collector dynamics, studies are continuously conducted in order to manage the heat production and enhance the collector efficiency. The aim is to design efficient controllers able to achieve the control objectives using the available boundary measurements. Indeed, only the inlet and the outlet temperatures can be measured. Thereafter, for an efficient control design with the available feedback information, controllers may require an estimated profile of the temperature along the collector tube. This motivates the design of observers able to ensure a stable estimation error within a relatively short transient time with respect to the system dynamics despite the varying external conditions. Considering their distributed nature, solar collectors are modeled by a first order hyperbolic partial differential equation (PDE) to describe the heat transport dynamics. However, the control design is usually based on approximate models using discretization numerical schemes (Camacho et al. (1997, 2007)). There is an abundant litterature on the design of observers for approximate discretized models while the design of observers for PDE systems has been less considered (Gallego and Camacho (2012); Gallego et al. (2013)).

Inspired by the backstepping observer introduced by Krstic et al. in (Krstic and Smyshlyaev (2008)) using the Volterra transformation, this paper presents a nonlinear observer for the estimation of the temperature profile along the concentrated distributed solar collector. The proposed observer is developed based on a nonlinear integral transformation considering a Luenberger form. The observation gain is obtained by stabilizing the estimation error in a finite time following stable transport dynamics. The proposed approach has the advantage of compromising between the simplicity of the implementation and the accuracy of the estimation. The performance of the proposed observer has been proven by numerical tests considering different working conditions starting from several initial conditions. The obtained results are promising and show good performance. The convergence of the estimated temperature is achieved gradually along the collector tube in a finite time. Moreover, the stabilization of the estimation error is ensured in a short transient time (seconds) compared to the system dynamics which is an interesting feature for the closed loop control.

The remainder of this paper is organized as follows. Section 2 describes the heat transport dynamics of the solar collector. Then, the estimation problem is formulated in section 3, followed by the observer design and proof of the estimation error convergence. Some numerical results are presented and discussed in section 4. Finally, section

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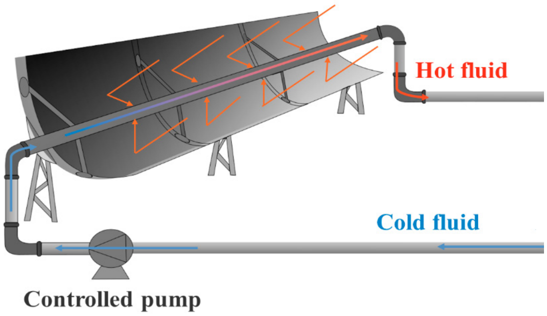

Fig. 1. Concentrated distributed solar collector

5 summarizes the obtained results and gives some future directions.

2. SYSTEM DESCRIPTION

A parabolic distributed solar collector is an industrial system producing thermal energy using heat transport in a thermal carrier fluid. The collector concentrates the received sunlight with its parabolic shaped mirrors to the central tube where the fluid flows (see Fig.1).

The parabolic collector is a spatially distributed system which is modeled by a first order hyperbolic PDE describing the conservation of the energy by considering the fluid temperature as the system state. Neglecting the heat losses and the thermal exchanges between the tube and the fluid, the model is expressed by (Camacho et al. (2007, 1997)):

$$\begin{cases} T_t(x,t) + \ u(t)\ T_x(x,t) \ = \ s(t), \\ T(0,t) \ = \ T_{in}(t), \\ y(t) = T(L,t) \ = \ T_{out}(t), \end{cases} \tag{1}$$

where $x \in [0, L]$ denotes the position along the pipe and $t \in \mathbb{R}^+$ represents the time. $T(x,t) \in \mathbb{R}^+$ refers to the fluid temperature at a certain position x and time t, where $T_t(x,t) \equiv \frac{\partial T(x,t)}{\partial t}$ and $T_x(x,t) \equiv \frac{\partial T(x,t)}{\partial x}$ are the first derivatives with respect to time and space respectively. $s(t) = \frac{\nu_0 G}{\rho c A_s} I(t)$ is the source term which depends on the solar irradiance $I(t)$. $u(t) = \frac{Q(t)}{A_s}$ is the system control input which is function of the fluid volumetric flow rate $Q(t)$. Besides, T_{in} and T_{out} denote the measured boundary values of the fluid temperature respectively at the positions $x = 0$ and L such that $y(t) = T_{out}$ is the system output. The remaining system parameters are summarized below.

ρ	Fluid density (kg m^{-3})
c	Fluid Specific heat capacity (J C^{-1} kg^{-1})
A_s	Tube cross-sectional area (m^2)
ν_0	Mirrors optical efficiency
G	Mirrors optical aperture (m)
L	Length of the collector tube (m)

Considering the system technology, the limited number of accessible measurement points along the collector makes the feedback control design constrained due to the insufficient information about the system dynamics. Thereafter, controllers may require the design of observers in order to estimate the temperature profile along the collector.

3. PROBLEM FORMULATION AND OBSERVER DESIGN

We consider the distributed model of the system dynamics defined by (1) with measured external disturbances. The goal is to estimate the distributed profile of the temperature along the collector tube for all admissible control inputs $u(t)$ and measured outputs $y(t)$.

The objective is to design an observer for the distributed system defined by (1) such that the state estimate vector $\hat{T}(x,t)$ converges in a finite time to the state vector $T(x,t)$.

Consider the following state observer:

$$\begin{cases} \hat{T}_t(x,t) + \ u(t)\ \hat{T}_x(x,t) \ = \ s(t) + L(x,t)(y(t) - \hat{y}(t)), \\ \hat{T}(0,t) \ = \ T_{in}(t), \\ \hat{y}(t) = \hat{T}(L,t), \end{cases} \tag{2}$$

such that $\hat{T}(x,t)$ and $\hat{y}(t)$ are the estimated state and output respectively. $L(x,t)$ is the observer gain obtained by forcing the estimated state $\hat{T}(x,t)$ to converge to the state vector $T(x,t)$ in a finite time for all admissible control inputs.

Define the estimation error $\tilde{T}(x,t)$ such that:

$$\tilde{T}(x,t) = T(x,t) - \hat{T}(x,t), \tag{3}$$

with the corresponding dynamics defined by:

$$\begin{cases} \tilde{T}_t(x,t) = -\ u(t)\ \tilde{T}_x(x,t) \ - L(x,t)\tilde{T}(L,t), \\ \tilde{T}(0,t) \ = \ 0, \end{cases} \tag{4}$$

Our objective is to force the estimation error dynamics defined in (4) to follow the stable target dynamics given by (Lemos et al. (2014)):

$$\begin{cases} w_t(x,t) = -\ u(t)\ w_x(x,t), \\ w(0,t) \ = \ 0, \end{cases} \tag{5}$$

in order to stabilize the estimation error $\tilde{T}(x,t)$ in a finite time and consequently force the estimated state $\hat{T}(x,t)$ to converge to the real state $T(x,t)$.

Proposition 1. The observer error system defined by (4) converges in a finite time, for all admissible control inputs $u(t)$, considering the following observation gain:

$$L(x,t) = u(t)G(x), \tag{6}$$

such that:

$$G(x) = -K(x,L), \tag{7}$$

where $K(x,\sigma)$ verifies the following nonlinear distributed equation, for $(x,\sigma) \in [0,L]^2$:

$$K_\sigma(x,\sigma) + K_x(x,\sigma) = K(x,L)K(L,\sigma), \tag{8}$$

subject to the boundary condition:

$$K(0,\sigma) = 0. \tag{9}$$

Proof. Consider the following transformation:

$$\tilde{T}(x,t) = w(x,t) - \int_0^L K(x,\sigma)w(\sigma,t)d\sigma. \tag{10}$$

Differentiating (10) with respect to time, we obtain:

$$\tilde{T}_t(x,t) = w_t(x,t) - \int_0^L K(x,\sigma)w_t(\sigma,t)d\sigma. \tag{11}$$