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A Numerical Implementation of an Extended Luenberger Observer for a Class of Semilinear Hyperbolic PIDEs

Andreas Deutschmann * **Lukas Jadachowski** ** **Andreas Kugi** *,**

* *Automation and Control Institute, Complex Dynamical Systems Group, Vienna University of Technology, Vienna, Austria*
** *Christian Doppler Laboratory for Model-Based Process Control in the Steel Industry, Vienna University of Technology, Vienna, Austria*

Abstract: In this paper, an extended Luenberger observer for a class of semilinear hyperbolic PIDEs is proposed to solve the problem of state estimation. The nonlinear observer error dynamics is linearized and then stabilized locally by applying the backstepping method for time-varying linear hyperbolic PIDEs presented recently. Since the resulting kernel equation is computationally quite expensive, a numerical algorithm is proposed to obtain the solution in an efficient way. Finally, this algorithm is applied to an example with multiple equilibrium points, and the influence of the design parameters on the stability is investigated in more detail.

Keywords: Distributed-parameter System, Boundary Observer, Backstepping, Hyperbolic PIDE, Extended Luenberger Observer, Numerical Implementation.

1. INTRODUCTION

The problem of state estimation is of quite general interest for both modern control schemes (see, e.g., Krstic and Smyshlyaev (2008) and Meurer and Kugi (2009)) as well as process monitoring and fault detection purposes (e.g., Aamo et al.). Since most technical processes exhibit some kind of nonlinear behaviour, the observer design for such systems is of high practical interest. While several solutions do exist for lumped-parameter systems (LPSs), the results are rather scarce for nonlinear distributed-parameter systems (DPSs) (e.g., for Volterra nonlinearities see Vazquez and Krstic (2008a,b)).

Thus, apart from the classical early-lumping approach, the observer error dynamics is typically simplified by singular perturbation theory (see, e.g., Vazquez and Krstic (2006)) or by linearization. The latter approach leads to so-called extended Luenberger observers (see Meurer (2013) and Jadachowski et al. (2014)) whose linear but time-varying error dynamics needs to be stabilized by a suitable method. Choosing the backstepping method (see Krstic and Smyshlyaev (2008)) essentially boils down to determine the integral kernel of a Volterra (backstepping) transformation.

This contribution addresses extended Luenberger observers for a class of semilinear hyperbolic partial integro-differential equations (PIDEs) constituting a nonlinear version of the class introduced in Krstic and Smyshlyaev (2008). Motivated by results on linear time-varying PIDEs recently presented in Deutschmann et al. (2016), we locally stabilize the semilinear observer error dynamics by using the backstepping method. Since the determination of the time-varying kernel function is computationally very expensive, an efficient algorithm along the lines of Jadachowski et al. (2012) is proposed.

The remainder of this paper is structured as follows: First, the extended Luenberger observer is defined and the linearized observer error dynamics are formulated in Section 2. Then, in Section 3, the linearized error system is stabilized utilizing a modified backstepping method to prescribe a desired target dynamics (see Deutschmann et al. (2016)). Section 4 presents the proposed algorithm to solve the kernel equations. In Section 5, the numerical algorithm is tested and an extended Luenberger observer is applied to plants with multiple equilibrium points.

2. EXTENDED LUENBERGER OBSERVER

In this paper semilinear plants of the type

$$x_t(z,t) = x_z(z,t) + a(x(z,t),z,t) + g(x(0,t),z,t) + \int_0^z f(z,\xi,t)\,\gamma(\xi,x(\xi,t))\,\mathrm{d}\xi \tag{1a}$$

with boundary and initial conditions

$$x(z,0) = x_0(z) \tag{1b}$$

$$x(1,t) = u(t) \tag{1c}$$

are considered. Here, $u(t)$ represents an external input and the system output is given on the opposite boundary

$$y(t) = x(0,t). \tag{1d}$$

The system (1) is defined on the domain $(z,t) \in \Omega = (0,1) \times \mathbb{R}^+$. The functions a, g, f and γ are supposed to be continuous in all variables and continuously differentiable in x and the input $u(t)$ is at least piece-wise continuous.

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Assumption 1. It is assumed that there exists a sufficiently smooth solution to the system (1).

An extended Luenberger observer (cf. Meurer (2013)) for (1) can be constructed in the form

$$\hat{x}_t(z,t) = \hat{x}_z(z,t) + a(\hat{x}(z,t),z,t) + g(\hat{x}(0,t),z,t) + p(z,t)(y(t)-\hat{y}(t)) + \int_0^z f(z,\xi,t)\,\gamma(\xi,\hat{x}(\xi,t))\,\mathrm{d}\xi \tag{2a}$$

with the observer's boundary and initial conditions

$$\hat{x}(z,0) = \hat{x}_0(z) \tag{2b}$$

$$\hat{x}(1,t) = u(t) \tag{2c}$$

and the output

$$\hat{y}(t) = \hat{x}(0,t). \tag{2d}$$

The dynamics of the observation error $e(z,t) = x(z,t) - \hat{x}(z,t)$ can be written as

$$\begin{aligned} e_t(z,t) &= e_z(z,t) + a(\hat{x}+e,z,t) - a(\hat{x},z,t) - p(z,t)e(0,t) \\ &\quad + g(\hat{x}(0,t)+e(0,t),z,t) - g(\hat{x}(0,t),z,t) \\ &\quad + (F\,(\hat{x}+e))(z,t) - (F\,\hat{x})(z,t) \end{aligned} \tag{3}$$

using (1) and (2) and introducing the nonlinear Volterra operator $F : L^2([0,1]) \to L^2([0,1])$ with

$$(F\,x)(z,t) = \int_0^z f(z,\xi,t)\,\gamma\big(\xi, x(\xi,t)\big)\,\mathrm{d}\xi. \tag{4}$$

It can be shown that its Fréchet derivative is given by

$$(DF(x)\,\chi)(z,t) = \int_0^z f(z,\xi,t)\,\gamma_x(\xi,x(\xi,t))\,\chi(\xi,t)\,\mathrm{d}\xi. \tag{5}$$

Thus, assuming the observation error $e(z,t)$ to be relatively small, the approximations

$$a(\hat{x}+e,z,t) \approx a(\hat{x},z,t) + a_x(\hat{x},z,t)\,e \tag{6}$$

$$g(\hat{x}+e,z,t) \approx g(\hat{x},z,t) + g_x(\hat{x},z,t)\,e \tag{7}$$

$$(F\,(\hat{x}+e))(z,t) \approx (F\,\hat{x})(z,t) + (DF(\hat{x})\,e)(z,t), \tag{8}$$

can be used to linearize the semilinear error dynamics (3) around the current state estimate $\hat{x}(z,t)$ yielding

$$e_t(z,t) = e_z(z,t) + \tilde{a}(z,t)e(z,t) + \tilde{p}(z,t)e(0,t) + \int_0^z \tilde{f}(z,\xi,t)\,e(\xi,t)\,\mathrm{d}\xi \tag{9}$$

with

$$\tilde{a}(z,t) = a_x(\hat{x}(z,t),z,t) \tag{10a}$$

$$\tilde{p}(z,t) = g_x(\hat{x}(0,t),z,t) - p(z,t) \tag{10b}$$

$$\tilde{f}(z,\xi,t) = f(z,\xi,t)\,\gamma_x(\xi,\hat{x}(\xi,t)). \tag{10c}$$

We may now proceed to stabilize the linearized error dynamics (9) by using the backstepping method.

3. LOCAL STABILIZATION OF THE ERROR DYNAMICS

Due to (10), the stabilization of the linearized error dynamics constitutes a time-varying problem presented recently in Deutschmann et al. (2016). Accordingly, to map (9) onto a predefined target system

$$w_t(z,t) = w_z(z,t) - \mu(z)\,w(z,t) - \int_0^z h(z,\xi)w(\xi,t)\mathrm{d}\xi \tag{11}$$

$$w(1,t) = 0 \tag{12}$$

with the design parameters $\mu(z)$ and $h(z,\xi)$, a modified backstepping transformation

$$e(z,t) = \alpha(z,t)\,w(z,t) - \int_0^z k(z,y,t)\,w(y,t)\,\mathrm{d}y \tag{13}$$

with the auxiliary function $\alpha(z,t)$ and the integral kernel $k(z,y,t)$ is used.

The target system (11) is exponentially stable if

$$\mu_{\inf} - h_{\sup} > 0 \tag{14}$$

where

$$\mu_{\inf} = \inf_{z\in[0,1]} \mu(z) \tag{15a}$$

$$h_{\sup} = \sup_{(z,y)\in\mathcal{T}} |h(z,y)| \tag{15b}$$

with $\mathcal{T} = \{(z,y) \in \mathbb{R}^2 | 0 < y < z < 1\}$. The norm $\|w(z,t)\|_{L^2}$ is then bounded by

$$\|w(z,t)\|_{L^2} \le \exp[-(\mu_{\inf} - h_{\sup})\,t\,]\,\|w(z,0)\|_{L^2}. \tag{16}$$

Differentiating (13) with respect to z and t and inserting the results into (9) yields the set of equations (cf. Deutschmann et al. (2016))

$$\begin{aligned} k_z(z,y,t) + k_y(z,y,t) - k_t(z,y,t) &= -\beta(z,y,t)k(z,y,t) \\ &\quad + \alpha(z,t)h(z,y) + \alpha(y,t)\tilde{f}(z,y,t) \\ &\quad - \int_y^z k(z,\xi,t)h(\xi,y) + \tilde{f}(z,\xi,t)k(\xi,y,t)\,\mathrm{d}\xi \end{aligned} \tag{17a}$$

$$\alpha_t(z,t) = \alpha_z(z,t) + \beta(z,z,t)\,\alpha(z,t) \tag{17b}$$

$$k(1,y,t) = 0 \tag{17c}$$

with $\beta(z,y,t) = \tilde{a}(z,t) + \mu(y)$. A boundary condition for $\alpha(z,t)$ compatible with (17b) is arbitrarily chosen to be

$$\alpha(1,t) = 1. \tag{17d}$$

The desired observer gain $p(z,t)$ is given by

$$p(z,t) = g_x(\hat{x}(z,t),z,t) - \frac{1}{\alpha(0,t)}\,k(z,0,t). \tag{17e}$$

Remark 1. The kernel equations (17) are implicitly coupled to the observer (2) by the definitions (10). Analyzing this coupling is still an open problem (cf. Meurer (2013)). It is therefore assumed that the coupled system is well-posed and exhibits sufficiently smooth solutions at least for small initial observer errors $e(z,0) = x_0(z) - \hat{x}_0(z)$.

The well-posedness of the (uncoupled) set of equations (17) for $\alpha(z,t)$ and $k(z,y,t)$ is shown in Deutschmann et al. (2016) by considering a boundary-value problem on $(z,y,t) \in \mathcal{T} \times \mathbb{R}$ that can be converted into an implicit integral equation and solved by successive approximation. While this procedure imposes rather mild conditions on the regularity of the time-varying terms for theoretical results, using finite approximations of the resulting series solution is clearly not advisable if one wants to obtain numerical solutions of $k(z,y,t)$ and $\alpha(z,t)$ in an efficient manner. On the one hand, the iterative structure of successive approximation is typically rather slow. On the other hand, its inherent coupling of space and time is difficult to handle for online calculations compared to forward time-marching algorithms. It seems much more promising to truncate the domain in time and consider an initial-boundary-value (IBV) problem instead. In Jadachowski et al. (2012), an efficient numerical method has been proposed for the case of time-varying parabolic PDEs. A similar approach will be applied to the considered hyperbolic PIDEs in the following section.

Remark 2. Imposing a boundary condition for α at $z = 0$ would be possible. However, in this case the solutions are propagating backwards in time (cf. method of characteristics) which contradicts the goal of forward time-marching