Automatica 96 (2018) 293–297

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Second-order min-consensus on switching topology[☆]

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ARTICLE INFO

Article history:
Received 1 November 2017
Received in revised form 27 March 2018
Accepted 10 June 2018

Keywords:
Min-consensus
Multi-agent system
Double-integrator dynamics
Jointly connected graph

ABSTRACT

The agent dynamics of most of the multi-agent systems in practice are second-order. However, there is no existing min-consensus protocol for such systems. In this paper, we deal with this problem by proposing the first second-order min-consensus protocol with provable convergence. It is not trivial to extend the min-consensus result for the first-order case to the second-order one. Under certain conditions, the proposed protocol can guarantee global asymptotic min-consensus, even for the case with jointly connected communication graphs. An illustrative example is presented to verify the theoretical results and the efficiency of the proposed protocol.

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1. Introduction

Consensus is a fundamental issue in distributed cooperative control of multi-agent systems and distributed computing, which has broad applications (Jin & Li, 2018; Ren, Chao, Bourgeous, Sorensen, & Chen, 2008; Schenato & Fiorentin, 2011; Xu, Liu, & Gong, 2011). Depending on the common value at which an agreement is reached, special cases of consensus include average consensus, max-consensus, and min-consensus (Cheng, Hou, Tan, & Wang, 2011; Macellari, Karayiannidis, & Dimarogonas, 2017; Olfati-Saber & Murray, 2004; Rezaee & Abdollahi, 2015). Manitara and Hadjicostis (2017) employed max-consensus and min-consensus protocols to realize distributed stopping for average consensus in digraphs. Iutzeler, Ciblat, and Jakubowicz (2012) proposed two max-consensus algorithms for estimating the maximum value in wireless sensor networks. Zhang, Tepedelenlioğlu, Banavar, and Spanias (2013) proposed a soft-max approach for reliable computation of the maximum value of local measurements over autonomous sensor networks by max-consensus, which was also extended to deal with the min-consensus problem.

Extensive results about how to design consensus protocols for different types of systems under different situations have been reported in recent years. Cortés (2008) proposed some protocols for achieving various types of consensus. Li, Duan, Chen, and Huang (2010) introduced a unified framework to address the consensus of multi-agent systems consisting of agents with linear dynamics and the synchronization of complex networks. Shi, Xia, and Johansson (2015) proposed a max–min consensus algorithm and proved its convergence. Manfredi and Angeli (2017) proposed necessary and sufficient conditions for consensus in nonlinear monotone networks with unilateral interactions, where a max-consensus protocol for a system with three agents was also proposed. However, the protocols proposed by Manfredi and Angeli (2017) and Shi et al. (2015) are for first-order systems. Abdessameud and Tayebi (2013) addressed the consensus problem of double-integrator multi-agent systems. Huang, Duan, and Chen (2016) proposed a novel distributed consensus protocol for second-order linear multi-agent systems with a directed communication topology. Results have also been reported about consensus protocol design based on optimization approaches (Movric & Lewis, 2014; Zhang & Li, 2017b, c), even-triggered mechanism (Ma, Wang, & Lam, 2017; Nowzari & Cortés, 2016), etc.

However, there is no existing min-consensus protocol for double-integrator multi-agent systems. On the one hand, from the literature review, one of the research directions in the consensus community is to extend protocols for multi-agent systems of low-order agent dynamics to those of second-order or even higher-order agent dynamics. In other words, the completeness of consensus theory requires the development of such protocols. On the other hand, in practice, a wide class of agents can be described by double-integrator dynamics, such as mobile robots and unmanned aerial vehicles (Ren & Atkins, 2007). Consider the altitude alignment problem of multiple unmanned aerial vehicles. When the alignment altitude is required to be the minimal initial altitude, then a min-consensus protocol is needed. The min-consensus

☆ This work is supported by the National Natural Science Foundation of China (with number 61401385), by Hong Kong Research Grants Council Early Career Scheme (with number 25214015), by Departmental General Research Fund of Hong Kong Polytechnic University (with number G.61.37.UA7L), and also by PolyU Central Research Grant (with number G-YBMU). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Julien M. Hendrickx under the direction of Editor Christos G. Cassandras.

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https://doi.org/10.1016/j.automatica.2018.07.009
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protocol for double-integrator multi-agent systems may also be helpful for multi-mobile-robot systems in certain rendezvous task (Su, Wang, & Chen, 2010). It is not a trivial work to extend the results for first-order min-consensus or max-consensus to higher-order cases due to the inherent nonlinearity of the problem. The problem becomes more complicated for the case with switching topology. It should be noted that max-consensus can be reached via a corresponding min-consensus protocol via a mapping or conversion for the state variables.

The main contribution of this paper is summarized as follows. To the best of our knowledge, this is the first time that a min-consensus protocol is proposed for the consensus of a second-order multi-agent system. The protocol is theoretically proven to guarantee global asymptotic min-consensus under mild conditions, and the communication graphs are allowed to be static undirected connected, time-varying undirected connected, and jointly connected.

2. Preliminary and problem formulation

In this section, some useful definitions about graph theory and the problem formation are given.

2.1. Preliminary

Some basic definitions in graph theory (Godsil & Royal, 2001) are used in this paper. Consider an undirected connected graph $\mathbb{G} = (\mathbb{V}, \mathbb{E})$ consisting of N nodes, where $\mathbb{V} = \{1, 2, \ldots, N\}$ and $\mathbb{E}$ denote the set of nodes and the set of edges in the graph, respectively. The value of node i in the graph is denoted by x_i. The edge connecting node $i \in \mathbb{V}$ and another node $j \in \mathbb{V}$ is denoted by $(i; j)$. The set of neighbors of node i is denoted by $\mathbb{N}(i) = \{j \mid (i; j) \in \mathbb{E}\}$. The union of a collection of graphs $\mathbb{G}_1 = (\mathbb{V}, \mathbb{E}_1)$, $\mathbb{G}_2 = (\mathbb{V}, \mathbb{E}_2)$, $\cdots$, $\mathbb{G}_m = (\mathbb{V}, \mathbb{E}_m)$, where m is a finite integer, with the same node set $\mathbb{V}$ is defined as $\hat{\mathbb{G}} = (\mathbb{V}, \bigcup_{i=1}^{m}\mathbb{E}_i)$. The collection $\mathbb{G}_1, \mathbb{G}_2, \cdots, \mathbb{G}_m$ is said to be jointly connected if the union graph $\hat{\mathbb{G}}$ is connected (Lin & Jia, 2011; Liu & Huang, 2018).

2.2. Problem formulation

Consider a group of agents whose communication topology is described by an undirected graph. In this paper, we are interested in the min-consensus problem of such a N-agent system with identical second-order dynamics described as follows:

$$\ddot{x}_i = u_i,\ i = 1, 2, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ are the state and input of the ith agent, respectively. N denotes the number of agents in the system. Specifically, we aim at designing a distributed protocol such that

$$\lim_{t\to+\infty} x_i(t) = \min_{i\in\mathbb{V}}\{x_i(0)\}, \forall i \in \mathbb{V}, \tag{2}$$

where $x_i(t)$ denotes the value of x_i at time instant t and $\mathbb{V} = \{1, 2, \ldots, N\}$ is the set of nodes. In other words, it is expected to design a fully distributed protocol such that the agents of the multi-agent system achieves asymptotic min-consensus. The distributed property makes the protocol suitable for large-scale systems.

3. Min-consensus under switching topology

In practical applications, the failure of communication between two agents in a multi-agent system may happen due to various reasons, such as the existence of obstacle, long distance between the agents making them out of communication region, fault of communication units, etc. (Olfati-Saber & Murray, 2004). On the other hand, the communication between two agents may be reestablished when, for example, the fault of communication units has been eliminated. These situations correspond to switching communication topologies or graphs. There is also the case that the graphs may be unconnected during the process for reaching consensus. A worse case is that the graph is disconnected at every time instant. In this section, we mainly present the results about the proposed min-consensus protocol in the case with jointly connected graphs.

3.1. Protocol

Let the communication graph of multi-agent system (1) at time instant t be denoted by $\mathbb{G}(t) = (\mathbb{V}, \mathbb{E}(t))$. By the definition of jointly connected graph, the communication graph can also be denoted by $\mathbb{G}_{\sigma(t)}$, where $\sigma(t)$ is called the switching signal and $\sigma(t) \in \{1, 2, \ldots, m\}$ with m denoting the number of different graphs. Let the set of neighbors of node i in graph $\mathbb{G}_{\sigma(t)}$ be denoted by $\mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)$, $\forall i \in \mathbb{V}$. Then, the proposed min-consensus protocol for multi-agent system (1) under jointly connected graphs $\mathbb{G}_{\sigma(t)}$ is described as

$$u_i = -2\dot{x}_i - x_i + \min_{j\in\{i\}\cup\mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)}(\dot{x}_j + x_j),\ \forall i \in \mathbb{V}. \tag{3}$$

The design of the proposed protocol is partially inspired by the existing min-consensus protocol (Zhang & Li, 2017a) for single-integrator multi-agent systems for which the state value is used in the min term. As a result, for double-integrator ones, we make an attempt by incorporating the time derivative of the state into the min term.

Remark 1. Evidently, protocol (3) is in the following form:

$$u_i = h(x_i, \dot{x}_i, \chi, \nu),\ i = 1, 2, \ldots, N,$$

where $\chi = \{x_j | j \in \mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)\}$ and $\nu = \{\dot{x}_j | j \in \mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)\}$, from which we can readily observe that it is fully distributed. In other words, the control action of each agent only depends on its own state and the states of its neighbor agents.

3.2. Theoretical analysis

In this subsection, we present theoretical analysis about the performance of the proposed min-consensus protocol.

Theorem 1. *Given that $\dot{x}_i(0) = 0$, $\forall i \in \mathbb{V}$, starting from any initial state $x_i(0)$, $\forall i \in \mathbb{V}$, the multi-agent system (1) defined on a set of jointly connected graphs $\mathbb{G}_{\sigma(t)} = \{\mathbb{V}, \mathbb{E}_{\sigma(t)}\}$ asymptotically achieves min-consensus with $\lim_{t\to+\infty} x_i(t) = \min_{j\in\mathbb{V}}\{x_j(0)\}$, $\forall i \in \mathbb{V}$, when protocol (3) is used.*

Proof. Substituting protocol (3) into system dynamics (1) yields the following closed-loop dynamics:

$$\ddot{x}_i = -2\dot{x}_i - x_i + \min_{j\in\{i\}\cup\mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)}(\dot{x}_j + x_j),\ i = 1, 2, \ldots, N. \tag{4}$$

Let $z_i = \dot{x}_i + x_i$. Then, (4) can be rewritten as the following cascaded system:

$$\dot{x}_i = -x_i + z_i, \tag{5a}$$

$$\dot{z}_i = -z_i + \min_{j\in\{i\}\cup\mathbb{N}_{\mathbb{G}_{\sigma(t)}}(i)}\{z_j\}, \tag{5b}$$

where $i = 1, 2, \ldots, N$.

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