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Brief paper

Stability and stabilization of periodic piecewise linear systems: A matrix polynomial approach[☆]

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In this paper, new conditions of stability and stabilization are proposed for periodic piecewise linear systems. A continuous Lyapunov function is constructed with a time-dependent homogeneous Lyapunov matrix polynomial. The exponential stability problem is studied first using square matricial representation and sum of squares form of homogeneous matrix polynomial. Constraints on the exponential order of each subsystem used in previous work are relaxed. State-feedback controllers with time-varying polynomial controller gain are designed to stabilize an unstable periodic piecewise system. The proposed stabilizing controller can be solved directly and effectively, which is applicable to more general situations than those previously covered. Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Periodic linear system is a class of systems which have periodic dynamics. It is easy to find various prototypes in various engineering applications, such as rotor-bearing systems and rotor–blade systems. Much attention of periodic systems has been put on their theoretical development and engineering applications (Bittanti & Colaneri, 2008; He, Han, & Wang, 2014; Jiao, Cai, & Li, 2016; Shao & Zhao, 2017; Tao, Lu, Su, Wu, & Xu, 2017). However, one may observe that the results of control problems for continuous-time periodic systems is not that rich when compared with those of discrete-time periodic systems. This is due to that discrete-time periodic systems can be converted to time-invariant systems with the lifting techniques; while for continuous-time periodic systems, the Floquet problem which helps obtain a constant dynamic matrix is considerably more difficult to solve. Some efforts of solving the control problems for continuous-time periodic linear system can be found in Zhou (2008), Zhou and Duan (2012), Zhou, Hou, and Duan (2013) and Zhou and Qian (2017). For more results about periodic systems, the reader may refer to Bittanti and Colaneri (2008) and its references.

Periodic piecewise linear system is a special kind of periodic linear systems, which consists of several time-invariant subsystems running periodically. Many engineering operations involving DC–DC converters and conveyor systems could be treated as periodic piecewise linear systems. From another aspect, after truncation and approximation, the periodic time-varying linear system may be described as periodic piecewise linear system (Farhang & Midha, 1995; Selstad & Farhang, 1996; Zhou & Qian, 2017). By exploiting the special dynamic characteristics, techniques targeted for periodic piecewise linear systems can be used to tackle the control problems of continuous-time periodic time-varying system. The stability of periodic time-varying system is investigated in Zhou and Qian (2017) with the periodic piecewise linear model approximation. The asymptotic stability, finite-gain L_p stability and uniformly boundedness are studied with frequency responses of the approximated periodic piecewise linear system. On the other hand, periodic piecewise linear systems can be treated as a special case of switched systems, of which the switching signal is periodic, and the switching sequence and dwell time of each subsystem is fixed. Techniques for switched systems (Xiang & Xiao, 2014; Zhai, Hu, Yasuda, & Michel, 2001; Zhao, Liu, Yin, & Li, 2014; Zhao, Zhang, Shi, & Liu, 2012) may therefore be used for periodic piecewise systems. The average dwell time approach is commonly adopted in the above results and it is also extended to the filtering problem of fuzzy switched systems with stochastic perturbation in Shi, Su, and Li (2016). The stabilization problem for discrete-time switched systems with additive disturbance is investigated in

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Zhang, Zhuang, Shi, and Zhu (2015) with quasi-time-varying Lyapunov function. It is known that the problem of time-dependent switching stabilization of switched systems composed of unstable subsystems is challenging, a novel idea of using invariant subspace analysis method to solve this issue is proposed in Zhao, Yin, and Niu (2015). With the techniques broadly used in switched systems, the exponential stability analysis of periodic piecewise linear systems can be found in Li, Lam, Chen, Cheung, and Niu (2015) and Li, Lam, and Chung (2015). A continuous time-varying Lyapunov function based on an interpolation formulation is adopted to investigate the stability problem in Li, Lam, Chen et al. (2015), for each subsystem, the Lyapunov function has its own varying rate in time. Furthermore, a discontinuous Lyapunov function is formulated in Li, Lam, and Chung (2015) to study the stability problem, which allows the Lyapunov function to have incremental bounds at the subsystem switching instants and, the Lyapunov matrix incremental bounds may be different when switching between different subsystems. It brings more slack variables and relaxes the constraints while understandably increases the computational burden and complexity. Moreover, some necessary and sufficient exponential stability conditions are also proposed in Li, Lam, and Chung (2015) based on the transition matrix, which greatly facilitates the stability verification of periodic piecewise systems. The stabilization problem of periodic piecewise system is studied in that work as well. Different controllers with constant controller gains are designed for each subsystem, and the corresponding algorithm is provided to solve the controller gain. The finite-time stability and stabilization problems of periodic piecewise are studied in Xie, Lam, and Li (2017) and a corresponding H_∞ controller is proposed in that work as well. The application of control on periodic piecewise system subject to actuator saturation can be found in Li, Lam, and Cheung (2016), where a controller is designed for periodic piecewise vibration system to attenuate the vibration.

In this work, new conditions on exponential stability and stabilization problems are investigated for periodic piecewise linear systems. Different from the previous works (Li, Lam, Chen et al., 2015; Li, Lam, & Chung, 2015), a nonlinear Lyapunov matrix polynomial is established instead of the linear interpolation Lyapunov matrix, which not only introduces more free variables but also helps relax the conditions with a more general class of Lyapunov matrices. A matrix polynomial method is used in the robustness analysis of system (Chesi, Garulli, Tesi, & Vicino, 2009), and it is also extended to switched systems with time-varying uncertainties in Briat (2015). In this work, the constructed Lyapunov matrix polynomial is continuous and time-dependent, techniques from Chesi et al. (2009) such as square matricial representation and sum of squares form are used to reformulate the Lyapunov matrix polynomial to derive the condition on exponential stability. Moreover, the constraints on some mode-dependent parameters are relaxed as well. Based on the proposed exponential stability condition, a stabilizing controller is designed as well. Comparing with the controller proposed in Li, Lam, and Chung (2015), apart from possibly lower conservatism, this controller can be solved directly with LMI conditions rather than through iterative algorithm. Moreover, controllers are also designed for unstabilizable subsystems, which may help stabilize the system more effectively and make the condition easier to solve. The paper is organized as follows. Definitions of exponential stability and matrix polynomials are provided in Section 2. In Section 3, the stability criterion for periodic piecewise linear systems with continuous time-dependent polynomial Lyapunov function are derived. Based on the result, time-varying state-feedback controllers are synthesized to stabilize the system in Section 4. Numerical examples are presented in Section 5 to demonstrate the merits of the proposed techniques and the work is concluded in Section 6.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{S}^n$ denotes a n-dimensional symmetric matrix. $\|\cdot\|$ stands for the Euclidean vector norm, the superscript $'$ refers to matrix transposition, $\overline{\lambda}(\cdot)$, $\underline{\lambda}(\cdot)$ stand for the maximum, minimum eigenvalues of a real symmetric matrix, respectively. In addition, $P > 0$ means that P is a real symmetric and positive definite matrix.

2. Preliminaries

Consider a continuous-time periodic piecewise linear system of the form

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \tag{1}$$

where $x(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^s$ are the state vector and control input, respectively. It has a fundamental period T_p, that is, $A(t) = A(t+T_p)$, $B(t) = B(t+T_p)$, for $t \geq 0$. Suppose the interval $[0, T_p)$ is partitioned into S subintervals $[t_{i-1}, t_i)$, $i \in \mathcal{N}$, $\mathcal{N} = \{1, 2, \ldots, S\}$, where $t_0 = 0$, $t_S = T_p$, $(A(t), B(t))$ is time-invariant under the ith subsystem and is given by (A_i, B_i). In other words, the dwell time for the ith subsystem (A_i, B_i) is $T_i = t_i - t_{i-1}$ with $\sum_{i=1}^{S} T_i = T_p$. In this case, system (1) is equivalently represented by

$$\dot{x}(t) = A_i x(t) + B_i u(t), \quad t \in [\ell T_p + t_{i-1}, \ell T_p + t_i) \tag{2}$$

where $\ell = 0, 1, \ldots,$ $i = 1, 2, \ldots, S$. A definition concerning the exponential stability of system (1) is given below.

Definition 1. Periodic piecewise system (1) with $u(t) = 0$ is said to be λ^*-exponentially stable if the solution of the system from $x(0)$ satisfies $\|x(t)\| \leq \kappa e^{-\lambda^* t}\|x(0)\|$, $\forall t \geq 0$, for some constants $\kappa \geq 1$, $\lambda^* > 0$.

In this work, a class of polynomial Lyapunov functions is adopted, some related definitions and techniques are introduced (Chesi, 2010; Chesi et al., 2009).

Definition 2 (*Monomial*). A function $f : \mathbb{R}^q \to \mathbb{R}$ is a monomial if

$$f(\tau) = c_a \tau^a$$

where $\tau \in \mathbb{R}^q$, $c_a \in \mathbb{R}$, $a \in \mathbb{N}^q$ and the quantity of $a_1 + \cdots + a_q$ is the degree of f.

Definition 3 (*Polynomial*). A function $p : \mathbb{R}^q \to \mathbb{R}$ is a polynomial if

$$p(\tau) = \sum_{i=1}^{q} f_i(\tau)$$

where $f_i(\tau)$, $i = 1, 2, \ldots, q$, is a monomial with finite degree, and the degree of p equals the largest degree of $f_1, f_2, \ldots, f_q$. The set of all p is denoted as $\mathcal{P} = \{p : \mathbb{R}^q \to \mathbb{R}\}$, and denote the set of all p of degree h as $\mathcal{P}_h$.

Definition 4 (*Homogeneous Polynomial*). A function $p : \mathbb{R}^q \to \mathbb{R}$ is a homogeneous polynomial of degree h in q scalars if

$$p \in \mathcal{P}_h.$$

It is interesting to observe that any polynomial of degree h can be viewed as a homogeneous polynomial with one more variable set to 1. In other words, $p(\tau) = \sum_{i=0}^{h} p_i(\tau)$ where $p_i \in \mathcal{P}_i$ can be expressed as a homogeneous polynomial of degree i with

$$p(\tau) = \hat{p}(y)|_{y_{q+1}=1}$$

where $y = (\tau', 1)'$ and $\hat{p}(y)$ is the homogeneous polynomial given as

$$\hat{p}(y) = \sum_{i=0}^{h} p_i(\tau) y_{q+1}^{h-i}.$$