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Brief paper

Dissipative observers for coupled diffusion–convection–reaction systems ☆

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ABSTRACT

The dissipativity-based observer design approach is extended to a class of coupled systems of 1-D semi-linear parabolic partial differential equations (PDEs) of diffusion–convection–reaction type with in-domain point measurements. This class of systems covers important application examples like tubular or catalytic reactors. By combining a dissipativity (sector) condition for the nonlinearity with a modal measurement injection for the linear differential operator sufficient conditions for the exponential convergence of the observer are derived in the form of a linear matrix inequality (LMI). The performance of the proposed approach is illustrated for an exothermic tubular reactor model.

1. Introduction

Diffusion–convection–reaction systems are described by semi-linear parabolic PDE models and are fundamental in applications where diffusion processes have to be considered explicitly. The problem of estimating the state profile on the basis of some local point-wise measurements is non-trivial. Given the nonlinear effects, phenomena like steady-state multiplicity or bifurcation behavior have to be considered and the unknown initial profile can lead to completely erroneous model predictions. Thus, adequate measurement injection mechanisms must be provided in combination with model predictions to design so-called observers, which ensure that the erroneous initial guess is compensated and the profile estimate converges to the actual state profile.

The design of observers for distributed-parameter systems follows either the early- or the late-lumping approach. In the early-lumping approach the system dynamics are first discretized in the spatial coordinate and then observer theory for finite-dimensional (higher order) systems is applied (see, e.g. Christofides, 2001 and the references therein). In the late-lumping approach the physical description based on PDEs is exploited and the resulting observer is itself a PDE, which is implemented by using suitable approximation techniques. This gives rise to the concept of first design then discretize. In the present study the late-lumping approach for the observer design for coupled 1-D semi-linear parabolic systems is addressed.

Generally speaking, in the last decades the observer design for distributed-parameter systems based on the late-lumping approach has reached important milestones. Particular approaches include modal designs (Curtain, 1982; Curtain & Zwart, 1995), back-stepping (Baccoli & Pisano, 2015; Jadachowski, Meurer, & Kugi, 2015; Krstic & Smyshlyaev, 2008; Meurer, 2013a, 2013b), dissipativity and matrix inequality based designs (Castillo, Witrant, Prieur, & Dugard, 2013; Hagen, 2006; Hagen & Mezic, 2003; Schaum, Moreno, & Alvarez, 2008; Schaum, Moreno, Fridman, & Alvarez, 2013; Schaum, Moreno, & Meurer, 2016; Yang & Dubljevic, 2014), adaptive observers for positive-real systems (Curtain, Demetriou, & Ito, 2003), as well as sliding-mode observers (Orlov, 2009). There are still some important open questions concerning the design of observers, in particular for coupled semi-linear PDE systems with in-domain measurements.

For finite-dimensional nonlinear systems dissipativity-based observer design (Moreno, 2005, 2008) provides an effective means for dealing with complex nonlinearities and yields explicit (local or non-local) convergence results in terms of (linear) matrix inequalities, which in turn can be solved using efficient numerical solvers. The dissipativity-based approach has been extended to scalar 1-D parabolic PDE systems with single in-domain Schaum et al. (2008)

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and boundary measurement (Schaum et al., 2013) as well as coupled diffusion–reaction systems with a single in-domain measurement (Schaum et al., 2016). In particular, in Schaum et al. (2008, 2016) the problem of injecting a single measurement into the semi-linear parabolic PDE is performed on the basis of a separation of the semi-linear dynamics into a linear dynamical and a nonlinear static part in the sense of the absolute stability analysis (Hagen, 2006; Khalil, 1996) followed by a modal observer design for the linear differential operator (see e.g. Curtain, 1982) to move the dominant eigenvalues sufficiently far into the left-half open complex plane to ensure that the potentially destabilizing effects of the nonlinearity are compensated.

In view of these preliminary results the purpose of the present study is to extend the dissipativity-based observer design to coupled 1-D semi-linear parabolic diffusion–convection–reaction systems with multiple in-domain measurements and input-dependent nonlinearities, using the modal measurement injection approach addressed in Curtain (1982) and Curtain and Zwart (1995) for linear systems and in Schaum et al. (2008, 2016) for semi-linear systems with a single in-domain measurement.

The paper is organized as follows. In Section 2 the observation problem is formulated. In Section 3 relevant concepts and results from dissipativity theory are recalled and established. In Section 4 the dissipativity-based modal-measurement injection observer design is developed, exponential convergence conditions are derived in form of an LMI and the approach is put into perspective with other design methods. In Section 5 the case of an exothermic tubular reactor model is discussed and numerical simulations are presented to illustrate the theoretical assessments. Conclusions are drawn in Section 6.

Notation:

- $\mathcal{H} = L^2(0,1) \cap \mathcal{C}^1(\mathbb{R}_+)$ denotes the space of real-valued functions $w(z,t) : [0,1] \times \mathbb{R}_+ \to L^2(0,1)$, which are differentiable with respect to t. $\mathcal{H}$ is an Hilbert space equipped with the inner product
$$(v, w)_{L^2} = \int_0^1 vw dz$$
inducing the standard L^2 norm $\|\cdot\|_{L^2} = \sqrt{(\cdot,\cdot)_{L^2}}$.
- The product space $\mathcal{H}^n$ is a real-valued Hilbert space with inner product
$$(\boldsymbol{v}, \boldsymbol{w}) = \int_0^1 \boldsymbol{v}^T \boldsymbol{w} dz = \sum_{j=1}^n \left(v_j, w_j\right)_{L^2}$$
and the induced norm $\|\cdot\| = \sqrt{(\cdot,\cdot)}$.
- The space $H^2(0,1)$ is the Sobolev space of functions with first and second derivative in $L^2(0,1)$.
- For a complex number $\lambda \in \mathbb{C}$, $\bar{\lambda}$ denotes its complex conjugate.
- $\sigma(A)$ denotes the spectrum of a matrix or an operator A.
- The dependence of functions on z and t will be denoted only when it is important for the readability.

2. Problem statement

In the following a system described by coupled semi-linear parabolic PDEs is considered with measurements in the interior of the spatial domain

$$\partial_t \boldsymbol{x} = \mathcal{A}\boldsymbol{x} + G\boldsymbol{\varphi}(\boldsymbol{\sigma}, \boldsymbol{y}, \boldsymbol{u}) + \boldsymbol{\chi}(\boldsymbol{y}, \boldsymbol{u}) \tag{1a}$$
$$\mathfrak{B}\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u}_b) \tag{1b}$$
$$\boldsymbol{\sigma} = H\boldsymbol{x} \tag{1c}$$
$$\boldsymbol{y} = C\boldsymbol{x} \tag{1d}$$

with initial condition $\boldsymbol{x}(z,0) = \boldsymbol{x}_0(z) \in \left(H^2(0,1)\right)^n$. In (1) $\boldsymbol{x}(z,t) \in \mathcal{H}^n$ is the state, $\mathcal{A} : \mathcal{H}^n \to \mathcal{H}^n$ is a parabolic differential operator, $G(z)$ is an $(n \times q)$-matrix with entries $G_{ij}(z) \in L^2(0,1)$, $\boldsymbol{\varphi}$ is a q-vector with entries $\varphi_i(z,t,\boldsymbol{\sigma}(z,t),\boldsymbol{y}(t),\boldsymbol{u}(z,t)) : [0,1] \times \mathbb{R}_+ \times \mathcal{H}^p \times \mathbb{R}^m \times \mathcal{U} \to \mathcal{H}$ which are once continuously differentiable in z, smooth in t and Lipschitz continuous in $\boldsymbol{\sigma} \in \mathcal{H}^p$, $\boldsymbol{y} \in \mathbb{R}^m$, $\boldsymbol{u} \in \mathcal{U}$, where $\boldsymbol{\sigma}$ is a combination of states defined according to the $(p \times n)$-matrix $H(z)$ with entries $H_{ij}(z) \in L^2([0,1])$, $\boldsymbol{y}$ is the measurement vector, $\boldsymbol{u}$ is a known input signal from the space of admissable domain inputs $\mathcal{U}$, and $\boldsymbol{\chi}(t,z,\boldsymbol{y},\boldsymbol{u})$ is a known function which is once continuously differentiable in z, smooth in t and Lipschitz continuous in $\boldsymbol{y}$ and $\boldsymbol{u}$. The linear boundary conditions are summarized in the operator $\mathfrak{B}$ with $\mathfrak{B}\boldsymbol{x}$ having entries $a_{i0}x_i(0,t) + b_{i0}\partial_z x_i(0,t)$ and $a_{i1}x_i(1,t) + b_{i,1}\partial_z x_i(1,t)$ for $i = 1,\ldots,n$. The function $\boldsymbol{f}(t,\boldsymbol{y}(t),\boldsymbol{u}_b(t)) \in \mathcal{C}^\infty\left(\mathbb{R}_+ \times \mathbb{R}^m \times \mathbb{R}^b, \mathbb{R}^{2n}\right)$ is an exogenous boundary input that is Lipschitz in $\boldsymbol{y}$ and $\boldsymbol{u}_b$, with $\boldsymbol{u}_b \in \mathcal{C}(\mathbb{R}_+,\mathbb{R}^p)$. The measurement vector $\boldsymbol{y}(t) \in \mathbb{R}^m$ is determined by the output operator $C : \mathcal{H}^n \to \mathbb{R}^m$ according to

$$\boldsymbol{y}(t) = C\boldsymbol{x}(z,t) = \begin{bmatrix} \mathbf{c}_1^T\left(\delta(z-\zeta_1), \boldsymbol{x}(z,t)\right) \\ \cdots \\ \mathbf{c}_m^T\left(\delta(z-\zeta_m), \boldsymbol{x}(z,t)\right) \end{bmatrix} \tag{2}$$

with $\delta(\zeta)$ denoting the Dirac δ-function centered at $z = \zeta$ and $\mathbf{c}_j^T \in \mathbb{R}^{1\times n}$, $j = 1,\ldots,m$.

Assumption 1. The operator $\mathcal{A}$ with the domain $\mathcal{D}(\mathcal{A}) = \{\boldsymbol{x} \in \left(H^2([0,1])\right)^n \mid \mathfrak{B}\boldsymbol{x} = \mathbf{0}\}$ is a Riesz spectral operator with real eigenvalues λ_j, $j \in \mathbb{N}$ fulfilling $0 > \lambda_1 \geq \lambda_2 \geq \cdots$ for which the algebraic and geometric multiplicities are the same, and whose eigenfunctions $\boldsymbol{\phi}_i$ together with the adjoint eigenfunctions $\boldsymbol{\psi}_i$ form a Riesz basis, i.e. $\left(\boldsymbol{\phi}_i, \boldsymbol{\psi}_j\right) = \delta_{ij}$, where δ_{ij} denotes the Kronecker δ function.

Remark 1. Note that the important class of linear (possibly unstable) diffusion–convection–reaction systems is covered in the class (1) as particular case with $H = G = I$, $\boldsymbol{\sigma} = \boldsymbol{x}$ and $\boldsymbol{\varphi}(\boldsymbol{x}) = K\boldsymbol{x}$ with a constant matrix $K \in \mathbb{R}^{n\times n}$.

The particular structure of (1) is motivated by the problem of Lur'e (Brogliato, Lozano, Maschke, & Egeland, 2007; Lur'e & Postnikov, 1944) considering the absolute stability of a linear system with input $\boldsymbol{\nu}$ and output $\boldsymbol{\sigma}$ endowed with a nonlinear feedback $\boldsymbol{\nu} = \boldsymbol{\varphi}(\boldsymbol{\sigma})$ satisfying a sector condition of the form

$$(\boldsymbol{\nu} - K_1\boldsymbol{\sigma})^T(K_2\boldsymbol{\sigma} - \boldsymbol{\nu}) \geq 0 \tag{3}$$

for appropriate matrices $K_1, K_2 \in \mathbb{R}^{q\times p}$. The problem addressed here consists in designing a measurement injection scheme with feedback gain $L(z)$ such that the distributed-parameter Luenberger observer

$$\partial_t \hat{\boldsymbol{x}} = \mathcal{A}\hat{\boldsymbol{x}} + G\boldsymbol{\varphi}(\hat{\boldsymbol{\sigma}}, \boldsymbol{y}, \boldsymbol{u}) + \boldsymbol{\chi}(\boldsymbol{y}, \boldsymbol{u}) - L(C\hat{\boldsymbol{x}} - \boldsymbol{y}) \tag{4a}$$
$$\mathfrak{B}\hat{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{y}, \boldsymbol{u}_b) \tag{4b}$$
$$\hat{\boldsymbol{\sigma}} = H\hat{\boldsymbol{x}} \tag{4c}$$

with initial condition $\hat{\boldsymbol{x}}(z,0) = \hat{\boldsymbol{x}}_0(z)$ yields a spatial–temporal estimate $\hat{\boldsymbol{x}}(z,t)$, which exponentially converges to the actual state, i.e. there exist positive constants $a, \gamma > 0$ such that

$$\left|\left|\hat{\boldsymbol{x}} - \boldsymbol{x}\right|\right| \leq a\left|\left|\hat{\boldsymbol{x}}_0 - \boldsymbol{x}_0\right|\right| e^{-\gamma t}. \tag{5}$$

3. Dissipativity concepts

For the purpose at hand consider the abstract formulation of a linear system

$$\begin{aligned} \partial_t \boldsymbol{x} &= \mathcal{A}\boldsymbol{x} + G\boldsymbol{\nu} \\ \mathfrak{B}\boldsymbol{x} &= \mathbf{0} \\ \boldsymbol{\sigma} &= H\boldsymbol{x} \end{aligned} \tag{6}$$