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Brief paper

Sliding mode predictive control of linear uncertain systems with delays☆

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ABSTRACT

In this paper, a continuous control strategy for robust stabilization of a class of uncertain multivariable linear systems with delays in both the state and control variables is proposed. A predictor is designed to compensate the delay effect in the control input, and then an integral sliding mode control technique along with super-twisting algorithm is applied to compensate partially the effect of the perturbation term. Finally, a nominal delay-free part of the control input is designed to stabilize the sliding mode dynamics. The proposed control scheme is extended to the class of systems modeled in Regular form. For this class of perturbed systems with delay in the state, a transformation to the systems with the delay-free state is proposed. The stability conditions of the closed-loop uncertain system are derived, and the results obtained in this work are compared against previous works. To show the effectiveness of the proposed method, simulation results are presented.

1. Introduction

Many modern industrial processes including systems where control signals are transmitted through networks, are modeled by delay differential equations. In these systems, the time delay can appear in the system state as well as in the control input. The last case is more dangerous for the closed-loop stability if the delay is large enough with respect to the plant dynamics rate and the standard memoryless feedback, i.e. the usual current system state, is used. Among stability analysis results for this case have been reported based on the Lyapunov–Krasovskii approach (see Fridman (2014), Mazenc, Niculescu, and Krstic (2012) and references therein). Basically, these results involve the upper bound on the time delay or the control gain values. On the other hand, the presence of the plant model uncertainty makes the situation more complex even for linear time invariant systems. A possible way to treat this problem is making use of the high gain or Sliding Mode (SM) control techniques (Utkin, Guldner, & Shi, 1999) which are effective tools to reject the system uncertainty. However, the direct implementation of these robust control techniques without taking into account delays may lead to oscillations and even instability of the closed-loop system (Fridman, Shustin, & Fridman, 1997). To overcome this problem, a predictor-based approach that enables to compensate for the time delay in the control input resulting in the delay-free closed-loop system, can be applied. First, the Smith predictor has been proposed in Smith (1957); however, this frequency domain approach can be implemented for open-loop stable systems only. To extend this approach to the general case of MIMO open-loop unstable systems, the Finite Spectrum Assignment approach based on the solution of LTI system has been proposed in Kwon and Pearson (1980) and Manitius and Olbrot (1979). The stability of the closed-loop system with predictor in absence of disturbances has been analyzed in Furukawa and Shimemura (1983), and an exhaustive analysis of predictive control scheme can be found also in Krstic (2009). In the presence of disturbances, the predictor was successfully applied to design SM controllers in Edwards and Spurgeon (1998), Polyakov (2012) and Roh and Oh (1999). However, the matching condition fulfilled for the uncertainties in the original system does not hold in the transformed delay-free (prediction) system with the conventional SM algorithm (Nguang, 2001). To solve this problem, the integral SM control (Utkin et al., 1999) implementation has been proposed in Loukianov, Espinosa-Guerra, Castillo-Toledo, and Utkin (2005, 2006). This approach allows to preserve the matching condition in the prediction system and, as a result, the unknown perturbation effect is reduced. On the other hand, in

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Léchappé, Moulay, Plestan, Glumineau, and Chriette (2015), a new predictive scheme has been designed permitting as well to reduce the perturbation effect.

In this paper, a class of linear perturbed systems with delay in the system state and the control input is considered, first, in general case and then in Regular form (Loukianov & Utkin, 1981). It is assumed that the time delay in the state vector is bigger than in the control input vector and the uncertainties satisfy the matching condition, as it is common in SM control design. A new predictive SM control scheme is designed using the advantages of the predictor (Léchappé et al., 2015) and the integral SM predictive controller (Loukianov et al., 2006). This scheme includes a predictor. The predictor is proposed in the form of Léchappé et al. (2015) to stabilize the SM dynamics. To design a stabilizing controller and achieve the robustness of the closed-loop predictive system where the matching condition is preserved contrary to the conventional SM controller (Polyakov, 2012; Roh & Oh, 1999), the integral SM predictive control technique (Loukianov et al., 2006) combined with super-twisting algorithm (Fridman & Levant, 2002; Moreno & Osorio, 2008) is used.

The stability analysis shows that in this case, even though the matching condition is preserved, the proposed control scheme cannot totally compensate for the unknown arbitrary perturbation. However, the perturbation effect can be reduced compared to Loukianov et al. (2006). Moreover, this scheme enables to totally compensate the unknown constant perturbation.

So, the proposed new predictive SM control scheme which reduces the matched disturbance effect in a linear system with delay in the state and input vectors can be considered as the main contribution of this paper.

The paper is organized as follows. In Section 2, the problem statement including assumptions is presented. The predictive control scheme is designed, first, for general class of linear systems in Section 3, and, then, for systems presented in Regular form in Section 4, including SM control design (Sections 3.1 and 4.1). The SM dynamics stability is analyzed in Section 3.2 and compared with the previous work in Section 3.3. To clarify the proposed control scheme, one example is presented in Section 4.2.

2. Problem statement

Consider an uncertain linear system with time delays described by

$$\dot{x}(t) = Ax(t) + Dx(t-\tau_0) + Bu(t-\tau_1) + f(x(t), x(t-\tau_0), t), \tag{1}$$

with the initial conditions given by, $x(t) = \varphi_0(t), \forall t \in [t_0 - \tau_0, t_0]$, $u(t) = \varphi_1(t)\ \forall t \in [t_0 - \tau_1, t_0]$, where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and control vectors, respectively; the unknown function $f \in \mathbb{R}^n$ represents model uncertainties including external disturbances; A, D and B are matrices of appropriate dimensions, rank $(B) = m$; τ_0 and τ_1 are time delays, $\tau_0 \geq \tau_1$.

In this paper, the objective is to design a robust controller in an uncertain scenario (1). Therefore, the following assumptions are required:

Assumption 1. The pair (A, B) is stabilizable, and the state x is available for the measurement.

Assumption 2. The unknown function $f(x(t), x(t-\tau_0), t)$ is locally Lipshitz and satisfies the matching condition (Drazenovic, 1969), namely, $f(x(t), x(t-\tau_0), t) = B\alpha(x(t), x(t-\tau_0), t)$; where function $\alpha(x(t), x(t-\tau_0), t), \alpha \in \mathbb{R}^m$ is bounded.

Assumption 3. There is a matrix $D_1 \in \mathbb{R}^{m\times n}$ such that $D = BD_1$ holds.

Assumption 4. All eigenvalues of matrices $M_1^{-1}M_2$ are located inside the open unit circle, where M_1 and M_2 are
$M_1 = B^T(I + e^{A\tau_1})B$ and $M_2 = B^T e^{A\tau_1} B$.

Assumption 5. The time delays τ_0 and τ_1 are constant and known.

3. Predictive control scheme for general case

3.1. SM control design

To eliminate the known delayed term $D_1 x(t-\tau_0)$ and robustly stabilize the system (1) under Assumption 3, the control law is redefined following the integral SM philosophy as

$$u(t) = u_0(t) + u_1(t) + u_2, \quad u_2(t) = -D_1 x(t-\Delta), \ \Delta = \tau_0 - \tau_1 \tag{2}$$

where $u_0(t) \in \mathbb{R}^m$ is the nominal control, and $u_1(t) \in \mathbb{R}^m$ will be designed to reject the perturbation term $\alpha(\cdot)$. Substituting (2) into (1), yields

$$\dot{x}(t) = Ax(t) + B[u_0(t-\tau_1) + u_1(t-\tau_1) + \alpha(x(t), x(t-\tau_0), t)]. \tag{3}$$

An integral sliding function $s_1(t) \in \mathbb{R}^m$ is formulated of the form

$$s_1(t) = Gx_p(t) + w(t), \ w(0) = -Gx_p(0), \tag{4}$$

with the predictive state $x_p(t) \in \mathbb{R}^n$ of the system (3) (the predictor is presented in Léchappé et al. (2015))

$$x_p(t) = \xi(t) + x(t) - \xi(t-\tau_1), \quad \xi(t) = e^{A\tau_1}x(t) + \int_{-\tau_1}^{0} e^{-A\theta} Bu_0(t+\theta)d\theta, \tag{5}$$

where $G \in \mathbb{R}^{m\times n}$ is a design matrix, and $w(t)$ is defined by

$$\begin{aligned}\dot{w}(t) = &- G[Ax_p(t) + Bu_0(t)] \\ &+ (GB + Ge^{A\tau_1}B)[u_1(t) - u_1(t-\tau_1)] \\ &- Ge^{A\tau_1}B[u_1(t-\tau_1) - u_1(t-2\tau_1)].\end{aligned} \tag{6}$$

Taking the time derivative of (4) and using (3), (5) and (6), yields

$$\begin{aligned}\dot{s}_1(t) = &(GB + Ge^{A\tau_1}B)[u_1(t) \\ &+ \alpha(x(t), x(t-\tau_0), t)] - Ge^{A\tau_1}B[u_1(t-\tau_1) \\ &+ \alpha(x(t-\tau_1), t-\tau_1, x(t-\tau_0-\tau_1), t-\tau_1)].\end{aligned} \tag{7}$$

To induce a sliding motion on $s_1(t) = 0$, the control component $u_1(t)$ is selected using super-twisting algorithm (Moreno & Osorio, 2008; Nagesh & Edwards, 2014) as

$$\begin{aligned}u_1(t) &= M_1^{-1}[-k_1\frac{s_1(t)}{\|s_1(t)\|^{\frac{1}{2}}} - k_2 s_1(t) + v(t) \\ &\quad + M_2 u_1(t-\tau_1)] \\ \dot{v}(t) &= -k_3\frac{s_1(t)}{\|s_1(t)\|} - k_4 s_1(t),\end{aligned} \tag{8}$$

where k_1, k_2, k_3 and k_4 are the control gains, $G = B^T$,

$$M_1 = B^TB + B^Te^{A\tau_1}B \text{ and } M_2 = B^Te^{A\tau_1}B. \tag{9}$$

Substituting control (8) into (7), results in

$$\begin{aligned}\dot{s}_1(t) &= -k_1\frac{s_1(t)}{\|s_1(t)\|^{\frac{1}{2}}} - k_2 s_1(t) + v(t) \\ &\quad + \Delta\alpha(x(t), x(t-\tau_0), t) \\ \dot{v}(t) &= -k_3\frac{s_1(t)}{\|s_1(t)\|} - k_4 s_1(t),\end{aligned} \tag{10}$$