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Scattering-based stabilization of non-planar conic systems[☆]

Anastasiia A. Usova, Ilia G. Polushin *, Rajni V. Patel

Department of Electrical and Computer Engineering, Western University, London, ON, N6A 5B9, Canada

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ABSTRACT

Methods for scattering-based stabilization of interconnections of nonlinear systems are developed for the case where the subsystems are non-planar conic. The notion of non-planar conicity is a generalization of the conicity notion to the case where the cone's center is a subspace with dimension greater than one. For a feedback interconnection of non-planar conic systems, a graph separation condition for finite-gain $\mathcal{L}_2$-stability is derived in terms of relationship between the maximal singular value of the product of projection operators onto the subsystems' central subspaces and the radii of the corresponding cones. Furthermore, a new generalized scattering transformation is developed that allows for rendering the dynamic characteristics of a non-planar conic system into an arbitrary prescribed cone with compatible dimensions. The new scattering transformation is subsequently applied to the problem of stabilization of interconnections of non-planar conic systems, with and without communication delays. Applications of the developed scattering-based stabilization methods to the problems of stable robot–environment interaction and bilateral teleoperation with multiple heterogeneous communication delays are discussed.

1. Introduction

Scattering transformation techniques have been used in the theory of electric networks, particularly transmission lines and networks with delays, since the middle of twentieth century (Ronald Wohlers, 1969). In the control systems area, applications of the scattering transformation can be traced back to work (Anderson, 1972) where a similar construction was used to establish relationship between passivity and small-gain theorems. In Anderson and Spong (1989), the scattering transformation was applied to the problem of stabilization of force reflecting teleoperators in the presence of communication delays. The latter work, together with parallel developments presented in Niemeyer and Slotine (1991), have made a very substantial impact on the bilateral teleoperation area, where the scattering-based stabilization is currently among the most popular methods to deal with instabilities caused by force reflection in the presence of communication delays (Hokayem & Spong, 2006; Niemeyer & Slotine, 2004; Nuño, Basañez, & Ortega, 2011; Secchi, Ferraguti, & Fantuzzi, 2016; Stramigioli, van der Schaft, Maschke, & Melchiorri, 2002; Sun, Naghdy, & Du, 2016). The stabilizing effect of the scattering transformation is based on the fact that a conventional scattering operator transforms a passive system into a system with $\mathcal{L}_2$-gain less than or equal to one (Anderson & Spong, 1989, Theorem 3.1). Assuming all involved subsystems are passive, scattering transformations implemented on both sides of a communication channel transform the corresponding subsystems into those with gain less than or equal to one; stability of the overall system then follows from the small-gain arguments.

Extensions of the scattering transformation techniques to the case of interconnections of not necessarily passive systems were recently proposed in Hirche, Matiakis, and Buss (2009) and Polushin (2014). These extensions are based on the observation that the conventional scattering transformation is essentially an operator of rotation by $\pi/4$ in the space of input–output variables. Introduction of more general scattering operators that include arbitrary rotations and input–output gains results in substantial generalizations of the scattering-based stabilization techniques. In particular, the methods developed in Polushin (2014) allow for stabilization of interconnections of arbitrary planar conic systems, with and without communication delays. The notion of a conic system was introduced and originally studied in 1960s by Zames (1966); extensions to the case of nonlinear conic sectors were subsequently developed in Safonov (1980) and Teel (1996). Conic

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* Corresponding author.
E-mail addresses: ausova@uwo.ca (A.A. Usova), ipolushi@uwo.ca (I.G. Polushin), rvpatel@uwo.ca (R.V. Patel).

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systems are nonlinear dynamical systems whose input–output behavior belongs to a dynamic cone. The notion of conicity studied in Zames (1966) was essentially planar in the sense that the dynamic cones were characterized by two scalar parameters which represent a conic sector on a plane. Even in this planar case, the notion of conicity is fairly general; in particular, it includes different versions of passivity, finite-gain $\mathcal{L}_2$-stability, *etc.*, as special cases. The stabilization methods developed in Polushin (2014) were based on a new generalized version of the scattering transformation which allows for rendering the dynamic input–output characteristics of an arbitrary planar conic system into a prescribed conic sector. Stability of interconnections can consequently be achieved by designing scattering transformation(s) that render the subsystems's cones in such a way that an appropriate stability condition (*i.e.*, a graph separation condition in the non-delayed case, or a small-gain condition in the presence of communication delays) is satisfied.

The class of planar conic systems, however, is quite limited in certain aspects. One particularly significant limitation is that, with the exception of systems with finite $\mathcal{L}_2$-gain, planar conic systems are required to have an equal number of inputs and outputs. The corresponding methods, including scattering-based design, are therefore limited to those systems where the number of inputs matches the number of outputs. Even in the latter case, description of a multi-input–multi-output system's cone in terms of two scalar parameters is typically overly crude; as a result, the methods that use such a parameterization lack flexibility, which in turn leads to limited applicability and analysis/design conservatism. Another substantial limitation of the planar conicity is that a feedback interconnection of two planar conic systems is, generally speaking, not a planar conic system; in fact, calculating a planar approximation of the corresponding dynamic cone can be a nontrivial task. The latter fact makes it difficult to use the notion of planar conicity for analysis of complex interconnections. All the above, in turn, limits the applicability of the existing scattering-based methods to stabilization of interconnections of general nonlinear systems.

In this paper, we develop an approach to scattering-based stabilization that removes all the limitations described above. The approach is based on an extension of the conicity notion to non-planar case, and subsequent development of a new generalized scattering transformation applicable to non-planar conic systems. The notion of non-planar conicity is based on an appropriate generalization of the planar conicity to the case where the cone's center is a subspace with dimension that can be greater than one. This generalization is quite substantial; in fact, the class of non-planar conic systems coincides with that of dissipative systems with quadratic supply rates (or (Q, S, R)-dissipative systems (Hill & Moylan, 1977)). In particular, for a given quadratic supply rate, the parameters of the corresponding non-planar cone can be calculated using the procedure presented below in Section 2.1. For a feedback interconnection of two non-planar conic systems, a graph separation condition for finite-gain $\mathcal{L}_2$-stability is derived in terms of relationship between the maximal singular value of the product of projection operators onto the subsystems' central subspaces and the radii of the corresponding cones. Subsequently, a new generalized scattering transformation is developed that allows for rendering the dynamic characteristics of a non-planar conic system into an arbitrary prescribed cone with compatible dimensions. This property of the new scattering transformation, in turn, allows for its effective use in the problem of stabilization of interconnections of non-planar conic systems, with and without communication delays. Applications of the developed scattering-based stabilization methods to the problems of stable robot–environment interaction and bilateral teleoperation with multiple heterogeneous communication delays are also described.

The paper has the following structure. In Section 2, the notion of non-planar conicity is introduced, and a procedure for calculation of the parameters of the (non-planar) dynamic cone for a dissipative system with a quadratic supply rate is described. In Section 3, a graph separation condition for finite-gain $\mathcal{L}_2$-stability of interconnection of two non-planar conic systems is presented. In Section 4, a new generalized scattering transformation is developed that allows for rendering the input–output dynamics of a non-planar conic system into an arbitrary prescribed cone. The scattering-based stabilization of interconnections of non-planar conic systems in the absence of communication delays is addressed in Section 5; application of this method to the problem of stable robot–environment interaction is discussed in Section 5.1. In Section 6, a scattering-based method for stabilization of non-planar conic systems' interconnections in the presence of multiple heterogeneous communication delays is developed; application of this method to bilateral teleoperation with communication delays is described in Section 6.1. Concluding remarks are given in Section 7. Preliminary versions of some of the results presented in Sections 2, 3 were reported in the conference paper (Usova, Polushin, & Patel, 2016), while preliminary versions of some of the results in Sections 4, 5 were presented in Usova, Polushin, and Patel (2017).

2. Non-planar conicity

Consider a nonlinear system of the form

$$\Sigma : \begin{cases} \dot{x} = f(x, \eta), \\ y = h(x, \eta), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $\eta \in \mathbb{R}^m$ the input, and $y \in \mathbb{R}^p$ the output of system (1). The functions $f(\cdot,\cdot)$, $h(\cdot,\cdot)$ are locally Lipschitz continuous in their arguments. A system (1) is said to be *dissipative* with respect to *supply rate* $w : \mathbb{R}^p \times \mathbb{R}^m \to \mathbb{R}$ if there exists a *storage function* $V : \mathbb{R}^n \to \mathbb{R}_+$ such that the inequality

$$V(x(t_1)) - V(x(t_0)) \le \int_{t_0}^{t_1} w(y(\tau), \eta(\tau))\, d\tau$$

holds along the trajectories of the system (1) for any $t_1 \ge t_0$, any initial state $x(t_0)$, and an arbitrary admissible control input $\eta(t)$, $t \in [t_0, t_1)$. In the definition below, $\mathbb{R}/\tilde{\pi}$ denotes the quotient set (*i.e.*, the set of equivalence classes) of $\mathbb{R}$ with respect to equivalence relation $\tilde{\pi} := \{\phi_1 \sim \phi_2 \text{ iff } \phi_1 - \phi_2 = k\pi,\ k \in \mathbb{Z}\}$, where $\mathbb{Z} := \{\dots, -1, 0, 1, \dots\}$ is the set of integer numbers.

Definition 1. A system Σ of the form (1) with $m = p$ is said to be (planar) interior conic with respect to the cone with center $\phi_c \in \mathbb{R}/\tilde{\pi}$ and radius $\phi_r \in (0, \pi/2)$ if it is dissipative with supply rate

$$w(y, \eta) = \begin{bmatrix} \eta^T & y^T \end{bmatrix} W(\phi_c, \phi_r) \begin{bmatrix} \eta^T & y^T \end{bmatrix}^T, \tag{2}$$

where matrix $W(\phi_c, \phi_r)$ is determined by the formula

$$W(\phi_c, \phi_r) := \frac{\lambda}{2} \cdot \begin{bmatrix} \cos 2\phi_c - \cos 2\phi_r & \sin 2\phi_c \\ \sin 2\phi_c & -\cos 2\phi_c - \cos 2\phi_r \end{bmatrix} \otimes \mathbb{I}_m, \tag{3}$$

where $\otimes$ denotes the Kronecker product, and $\lambda > 0$.

Representation (2), (3) of the supply rate of a conic system in terms of the cone's center ϕ_c and its radius ϕ_r is from Polushin (2014). There also exists a somewhat more conventional representation of the supply rate in terms of the cone's boundaries $a, b \in \mathbb{R} \cup \{\pm\infty\}$, $a \le b$, which was used for example in the classical work (Zames, 1966). Specifically, a system of the form (1) with $m = p$ is *interior* $[a, b]$-*conic* if it is dissipative with respect to the supply rate

$$w(y, \eta) := (b\eta - y)^T (y - a\eta). \tag{4}$$