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Distributed extremum-seeking control over networks of dynamically coupled unstable dynamic agents[☆]

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ABSTRACT

In this paper, a distributed extremum seeking control technique is proposed to solve a class of real-time optimization problems over a network of dynamic agents with unknown unstable dynamics. Each dynamic agent measures a cost that is shared over a network. A dynamic average consensus approach is used to provide each agent with an estimate of the total network cost. The extremum seeking controller operates at each agent to allow each agent to contribute to the optimization of the total cost, in a cooperative fashion. The extremum seeking control technique is based on a proportional–integral approach that provides improvements in transient performance over standard techniques. The contribution of the proposed technique is to solve the simultaneous stabilization and real-time optimization. A dynamic network simulation example is presented to demonstrate the effectiveness of the technique.

1. Introduction

Real-Time Optimization (RTO) is a process automation technology whose objective is to predict the economically optimal process operating policy in the near term. Extremum-seeking control (ESC) is an RTO control mechanism that is applicable when no exact model description is available, but where the objective function to be optimized is available from process measurements. This approach, which dates back to the 1920s (Leblanc, 1922), provides a mechanism by which a system can be driven to the optimum of a measured variable of interest (Tan, Moase, Manzie, Nešić, & Mareels, 2010). Many researchers have considered various ESC approaches over the last years (see Adetola and Guay (2007), Fu and Özgüner (2011), Ghaffari, Krstić, and Nešić (2012), Guay and Dochain (2013), Krstić (2000), Krstić and Wang (2000), Moase and Manzie (2012), Moase, Manzie, and Brear (2010), Nešić, Mohammadi, and Manzie (2010), Tan, Nešić, and Mareels (2006), Zhang and Ordóñez (2009)).

When dealing with complex dynamical systems, it is generally recognized that overall process objectives are difficult to achieve due to the computational complexity associated with centralized approaches. Thus, a decentralized or a distributed optimization approach is usually favoured in large-scale RTO systems design. In this approach, global process objectives are achieved by solving several local RTO subproblems. The distributed optimization task is said to be non-cooperative when each local RTO achieves its local optimization objectives. Non-cooperative RTO problems have been tackled using ESC by several researchers (Frihauf, Krstić, & Başar, 2011, 2012; Ghods, Frihauf, & Krstić, 2010; Stankovic & Stipanović, 2009). A Nash-seeking technique for the design of distributed optimization system is presented in Kutadinata, Moase, and Manzie (2015). This approach proposes a methodology for the design of dither signals in large networks. The distributed optimization task can also be cooperative when the local RTOs coordinate actions to optimize the sum of their assigned costs. A particular class of distributed cooperative optimizations has been the subject of several studies (Bertsekas & J.Tsitsiklis, 1989; Johansson, Rabi, & Johansson, 2009; Nedić & Ozdaglar, 2009). For a class of unconstrained optimization problems, it is shown in Nedić and Ozdaglar (2009) that it is possible to achieve overall system objectives by solving local problems and communicating the optimization results via the network. Few ESC techniques have been proposed to solve decentralized and distributed optimization problems (Kvaternik & Patel, 2012; Li, Qu, & Ingram, 2011; Poveda & Quijano, 2013). For constrained optimization problems, the Alternating Direct Method

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of Multipliers (ADMM) can be used to solve distributed and coordinated optimization problems (Boyd, Parikh, Chu, Peleato, & Eckstein, 2011; Schizas, Ribeiro, & Giannakis, 2008). A constrained distributed optimization approach was presented in Nedić, Ozdaglar, and Parrilo (2010) where a projection operator approach is used. The approach can effectively solve distributed optimization problems subject to a known computable projection to a known convex set. Extensions of cooperative optimization techniques to more general network architectures have been proposed in Gharesifard and Cortés (2014) and Nedić and Olshevsky (2013).

The control of networks of multi-agent systems with unknown dynamics has been treated in a number of studies. Lu and Chen (2005) have presented a consensus approach for a class of time-varying dynamical networks with unknown stable, decoupled system dynamics and first order agent dynamics with dynamic coupling. An extension to networks of second-order systems with unknown, decoupled, agent dynamics was presented in Su, Chen, Wang, and Lin (2011). Finite-time consensus over networks of unknown stable, decoupled, first-order agent dynamics was proposed in Cao and Ren (2014). Chen, Li, Ren, and Wen (2014) have used functional approximations to develop an adaptive consensus for multi-agents with unknown, but identical, control directions. An adaptive distributed synchronization approach was proposed in Das and Lewis (2010) for a class of multi-agent systems with unknown and nonidentical dynamics subject to disturbances. Adaptive consensus for systems with unknown higher-order decoupled dynamics was presented in Zhang and Lewis (2012). Agent dynamics with heterogeneous matching uncertainties were treated in Li, Duan, and Lewis (2014). In general, existing design approaches for multi-agent systems with unknown agent dynamics are limited to multi-agent dynamics where the only dynamic coupling arises from consensus or communication protocols.

This study proposes the design of a method of distributed optimization over networks of dynamic agents with unknown coupled unstable dynamics. The network dynamics are described by a large-scale unknown unstable nonlinear dynamical system operating over a local actuator and sensor communication network. Each agent has access to a local sensor measurement and a certain number of actuators. It is also able to communicate its sensor information with neighbouring agents. The local input–output dynamics of each agent are assumed to be unknown and can be affected by actuator variables and state variables from the network dynamics. To the best of our knowledge this class of multi-agent dynamical systems has not been treated in the literature. A distributed extremum-seeking controller is proposed to solve the optimization problem. This paper proposes a proportional–integral ESC (PIESC) design technique, initially proposed in Guay and Dochain (2014), that is implemented in a distributed environment to design cooperative systems to solve a distributed optimization problem over networks of unknown unstable dynamic agents. The main contribution of this paper is to show that the distributed PIESC can be effectively applied to the design of real-time optimization control systems that can stabilize the network of unstable dynamics to the unknown optimum of the total plant cost.

This paper is organized as follows. The problem is formulated in Section 2. In Section 3, the distributed ESC control system is presented. A simulation example is presented in Section 4. We conclude in Section 5.

2. Problem description

We consider a network of nonlinear systems of the form:

$$\dot{x}_i = f_i(x, \xi) + g_i(x, \xi)u \tag{1}$$

$$y_i = h_i(x) \tag{2}$$

where $x = [x_1^T, \ldots, x_p^T]^T \in \mathbb{R}^n$ is the vector of state variables, $u \in \mathbb{R}^m$ is the vector of input variables for the entire network, and $\xi \in \mathbb{R}^q$ is a vector of global variables, assumed to be unknown and unmeasured. Each x_i has dimension n_i with $\sum_{i=1}^p n_i = n$. The dynamics of each agent $i \in \{1, \ldots, p\}$ are governed by the dynamics (1) with local cost (2). It is assumed that each agent can only manipulate the local input variables $u_i \in \mathbb{R}^{m_i}$, with $\sum_{i=1}^p m_i = m$. The sets of inputs available at each node are assumed to be mutually exclusive. It is assumed that the vector fields $f_i(x, \xi) \in \mathbb{R}^{n_i}$ and $g_i(x, \xi) \in \mathbb{R}^{n_i \times m}$ are unknown smooth vector valued functions of x and ξ and that the functions $h_i(x)$ are unknown smooth functions of x.

The overall network cost function is the sum of all the individual costs:

$$J(x) = \sum_{i=1}^{p} h_i(x). \tag{3}$$

The objective is to steer the system to the equilibrium x^* and u^* that achieves the minimum value of $Y = J(x)$ using only measurements of the local cost and a communication network between agents.

The total network dynamics can be written in the form:

$$\dot{\xi} = \psi(x, \xi) \tag{4}$$

$$\dot{x} = f(x, \xi) + G(x, \xi)u \tag{5}$$

with global cost

$$Y = J(x). \tag{6}$$

Note that ξ represents the zero dynamics of the system, and for any nonlinear system there exists an appropriate state diffeomorphism which converts the system into the form (4)–(5). The drift vector field $f(x, \xi)$ and the control vector fields, $G(x, \xi)$, are such that:

$$f(x, \xi) = \begin{bmatrix} f_1(x, \xi) \\ \vdots \\ f_p(x, \xi) \end{bmatrix} \qquad G(x, \xi) = \begin{bmatrix} g_1(x, \xi) \\ \vdots \\ g_p(x, \xi) \end{bmatrix}$$

where $G(x, \xi)$ is a matrix valued function. The dynamics of the global variables ξ are described by the unknown vector field $\psi(x, \xi) : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^q$ which is assumed to be a smooth function of its arguments.

The equilibrium (or steady-state) network map is the vector $(x, \xi) = (\pi_x(u), \pi_\xi(u))$ that solves the following equations:

$$\psi(\pi_\xi(u), \pi_x(u)) = 0$$
$$f(\pi_\xi(u), \pi_x(u)) + G(\pi_\xi(u), \pi_x(u))u = 0.$$

The corresponding equilibrium cost function is given by:

$$Y_e = J(\pi_x(u)) = \ell(u)$$

where $\ell = J \circ \pi_x$. At equilibrium, the problem is reduced to finding the minimizer u^* of $Y_e = \ell(u)$.

In the following, we let $\mathcal{D}(u^*) \subseteq \mathbb{R}^{n+q}$ represent a neighbourhood of the equilibrium $(x, \xi) = (\pi_x(u^*), \pi_\xi(u^*))$ and we let $\mathcal{U} \in \mathbb{R}^m$ be a neighbourhood of u^*.

Assumption 1. For all $u \in \mathcal{U}$, there exist equilibrium values $x_e = \pi_x(u)$ and $\xi_e = \pi_\xi(u)$ such that $(x_e, \xi_e) \in \mathcal{D}(u^*)$.

The following assumption concerning the steady-state cost function $\ell(u)$ is required.

Assumption 2. The equilibrium steady-state map $\ell(u)$ is such that:

(1) there exists a set $\bar{\mathcal{U}} \subset \mathcal{U}$ with $u^* \in \bar{\mathcal{U}}$ such that $\ell(u)$ is continuously differentiable $\forall u \in \bar{\mathcal{U}}$.