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Brief paper

Dynamic output feedback stabilization of switched linear systems with delay via a trajectory based approach☆

Saeed Ahmed [a,*], Frédéric Mazenc [b], Hitay Özbay [a]

[a] Department of Electrical and Electronics Engineering, Bilkent University, Ankara 06800, Turkey
[b] Inria, Laboratoire des Signaux et Systèmes (L2S, UMR CNRS 8506), CNRS, CentraleSupélec, Université Paris-Sud, 3 rue Joliot Curie, 91192, Gif-sur-Yvette, France

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ABSTRACT

A new technique is proposed to construct observers and to achieve output feedback stabilization of a class of continuous-time switched linear systems with a time-varying delay in the output. The delay is a piecewise continuous bounded function of time and no constraint is imposed on the delay derivative. For stability analysis, an extension of a recent trajectory based approach is used; this is fundamentally different from classical Lyapunov function based methods. A stability condition is given in terms of the upper bound on the time-varying delay to ensure global uniform exponential stability of the switched feedback system. The main result applies in cases where some of the subsystems of the switched system are not stabilizable and not detectable.

1. Introduction

Switched systems have extensive applications in networks, automotive control, power systems, aircraft and air traffic control, process control, mechanical systems, and many other domains; see Lin and Antsaklis (2009), and the references therein. Due to this strong motivation, many questions related to switched systems such as stability (Liberzon, 2003; Liberzon & Morse, 1999; Sun & Ge, 2011), controllability (Liu, Lin, & Chen, 2013; Sun, Ge, & Lee, 2002), observability and reachability (Hespanha, Liberzon, Angeli, & Sontag, 2005; Ji, Feng, & Guo, 2007; Sun et al., 2002; Tanwani, Shim, & Liberzon, 2013), and synthesis (Pettersson, 2003; Sun & Ge, 2005), have been extensively studied in various contributions. Stability and stabilization are challenging problems pertaining to switched systems due to their hybrid nature and they are the main topic of the present paper.

There are mainly two approaches used in the literature for establishing the stability of switched systems:

(i) It is shown in Liberzon and Morse (1999) that existence of a common strict Lyapunov function is a necessary and sufficient condition for the switched system to be stable under arbitrary switching. On the other hand, when such a Lyapunov function exists, finding it may be a difficult task because it is an NP-hard problem; see Blondel and Tsitsiklis (1997). (ii) Liberzon and Morse (1999) also showed that even if a switched system does not possess a common strict Lyapunov function, it may be stable under a dwell-time requirement, typically derived using multiple strict Lyapunov functions. It is worth mentioning that multiple Lyapunov functions may lead to an undesirable attenuation property which can only be mitigated by imposing some strong assumptions; see Zhai, Hu, Yasuda, and Michel (2001).

Both of the above mentioned approaches are mainly developed for non-delayed systems. But measurement delays are present in many practical applications, such as chemical processes, aerodynamics and communication networks, and they are time-varying (see for instance Wu and Grigoriadis (2001) and Yan and Özbay (2005)). Therefore, the problem of stabilizing switched systems when a time-varying delay is present in the output is strongly motivated. State feedback stabilization of delayed switched linear systems is proposed in Vu and Morgansen (2010) using a combination of the multiple Lyapunov functions approach and the merging switching signal technique. An online and offline state feedback controller design for delayed switched linear systems in the detection of the switching signal are discussed in Xie and Wang (2005). Moreover, Koru, Delibaşı, and Özbay (2018) and Yan, Özbay, and Şansal (2014) present state

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* Corresponding author.
E-mail addresses: saeed.ahmed@bilkent.edu.tr (S. Ahmed), frederic.mazenc@l2s.centralesupelec.fr (F. Mazenc), hitay@bilkent.edu.tr (H. Özbay).

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feedback designs for delayed switched systems using a dwell-time based stability analysis approach. Note that Koru et al. (2018), Vu and Morgansen (2010), Xie and Wang (2005) and Yan et al. (2014) assume that all of the subsystems of the switched system are controllable. Finally, a state feedback stabilization problem for a class of delayed switched systems is studied in Kim, Campbell, and Liu (2006) and Sun, Wang, Liu, and Zhao (2008) under the assumption that the subsystems satisfy a certain Hurwitz convex combination condition. A common Lyapunov function approach is used in Kim et al. (2006) and Sun et al. (2008) to carry out stability analysis.

Contributions of this study: We propose a new technique to design observers and stabilizing dynamic output feedbacks offering robust stability results with respect to the presence of a time-varying pointwise delay in the output of the switched linear system. To establish the stability of the closed-loop switched system, we develop an extension of the trajectory based stability result recently proposed in Mazenc and Malisoff (2015), and Mazenc, Malisoff, and Niculescu (2017). We wish to point out that the new extension of the trajectory based approach we state and prove in the present paper is of interest by itself: it can be applied to a wide range of systems, notably to families of systems with time-varying delays wider than those invoked in Mazenc and Malisoff (2015), and Mazenc, Malisoff et al. (2017), and therefore it is one of the important contributions of our work.

We think that our main result can be regarded as an extension of Kim et al. (2006), Koru et al. (2018), Sun et al. (2008), Vu and Morgansen (2010), Xie and Wang (2005), Yan et al. (2014) and Zhai, Hu, Yasuda, and Michel (2000), offering new advantages because, (i) our study does not assume that all the states are available for feedback, (ii) it is not limited to systems whose all subsystems are stabilizable and detectable, (iii) we use a new extension of trajectory based approach for stability analysis which circumvents the serious obstacle presented by the search for appropriate Lyapunov functions, (iv) the application of our results is not restricted to the class of delayed switched systems where all the convex combinations of the subsystems in the absence of control must be Hurwitz, (v) we allow the delay to be time-varying and piecewise continuous function of time, and we do not impose any constraint on the upper bound of the delay derivative.

Now, we point out that the present paper is a continuation of our conference paper (Mazenc, Ahmed, & Özbay, 2017). We propose a significant extension of it by including, (i) dynamic output feedback stabilization, (ii) a new extension of trajectory based approach of Mazenc and Malisoff (2015) to produce less conservative results, (iii) a systematic way to compute an explicit value for the lower bound on the largest admissible delay for a broad family of switched systems so that when the delay is smaller than this bound, global uniform exponential stability (GUES) of the feedback switched systems is guaranteed. Moreover, we do not assume that the systems have synchronous switching sequences.

Organization of the paper: An extension of the trajectory based approach is given in Section 2. Section 3 is devoted to the main result of the paper. Section 4 discusses computational issues related to the delay bound. The results are illustrated by a numerical example in Section 5. Finally, we summarize and highlight our contributions in Section 6.

Notation: The notation will be simplified whenever no confusion can arise from the context. I denotes the identity matrix of any dimension. The usual Euclidean norm of vectors, and the induced norm of matrices, are denoted by $|\cdot|$. Given any constant $\tau > 0$, we let $C([-\tau, 0], \mathbb{R}^n)$ denote the set of all continuous $\mathbb{R}^n$-valued functions that are defined on $[-\tau, 0]$. We abbreviate this set as C_{in}, and call it the set of all *initial functions*. Also, for any continuous function $x : [-\tau, \infty) \to \mathbb{R}^n$ and all $t \geq 0$, we define x_t by $x_t(\theta) = x(t+\theta)$ for all $\theta \in [-\tau, 0]$, i.e., $x_t \in C_{\text{in}}$ is the translation operator. A vector or a matrix is nonnegative (resp. positive) if all of its entries are nonnegative (resp. positive). We write $M \succ 0$ (resp. $M \preceq 0$) to indicate that M is a symmetric positive definite (resp. negative semi-definite) matrix. For two vectors $V = (v_1 \dots v_n)^\top$ and $U = (u_1 \dots u_n)^\top$, we write $V \leq U$ to indicate that for all $i \in \{1, \dots, n\}$, $v_i \leq u_i$.

2. Extension of the trajectory based approach

We now provide with an extension of the trajectory based approach given in Mazenc and Malisoff (2015).

Lemma 1. *Let us consider a constant $T > 0$ and l functions $z_g : [-T, +\infty) \to [0, +\infty)$, $g = 1, \dots, l$. Let $Z(t) = (z_1(t) \dots z_l(t))^\top$ and, for any $\theta \geq 0$ and $t \geq \theta$, define $\mathfrak{V}_\theta(t) = \left(\sup_{s\in[t-\theta,t]} z_1(s) \dots \sup_{s\in[t-\theta,t]} z_l(s)\right)^\top$. Let $\Upsilon \in \mathbb{R}^{l\times l}$ be a nonnegative Schur stable matrix. If for all $t \geq 0$, the inequalities $Z(t) \leq \Upsilon \mathfrak{V}_T(t)$ are satisfied, then $\lim_{t\to+\infty} z_g(t) = 0 \quad \forall\, g = 1, \dots, l$.*

Proof. Since Υ is Schur stable, there is an integer $q > 1$ such that

$$|\Upsilon^q|\sqrt{l} < 1. \tag{1}$$

From Lemma 4 of Appendix A, we deduce that

$$Z(t) \leq \Upsilon^q \mathfrak{V}_{qT}(t) \tag{2}$$

for all $t \geq qT$. Consequently, $|Z(t)| \leq |\Upsilon^q||\mathfrak{V}_{qT}(t)|$.

Using $|\mathfrak{V}_{qT}(t)| \leq \sqrt{l} \sup_{s\in[t-qT,t]} |Z(s)|$, we obtain

$$|Z(t)| \leq |\Upsilon^q|\sqrt{l} \sup_{s\in[t-qT,t]} |Z(s)| .$$

This inequality, in combination with the inequality (1) and Mazenc and Malisoff (2015, Lemma 1), allows us to conclude the result. □

3. Observer and control design

We introduce a range dwell-time condition, i.e. a sequence of real numbers t_k such that there are two positive constants $\underline{\delta}$ and $\overline{\delta}$ such that $t_0 = 0$ and for all $k \in \mathbb{Z}_{\geq 0}$,

$$t_{k+1} - t_k \in [\underline{\delta}, \overline{\delta}]. \tag{3}$$

Definition 1. Let $\pi = \{(i_0, t_0), \dots, (i_k, t_k), \dots, | i_k \in \Xi,\ k \in \mathbb{Z}_{\geq 0}\}$ be a switching sequence. The function $\sigma : [0, \infty) \to \Xi = \{1, \dots, n\}$ such that $\sigma(t) = i_k$ when $t \in [t_k, t_{k+1})$ is called an associated *switching signal*.

We consider the continuous-time switched linear system:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) \\ y(t) = C_{\sigma(t)}x(t-\tau(t)) \end{cases} \tag{4}$$

with $x \in \mathbb{R}^{d_x}$, $u \in \mathbb{R}^{d_u}$, $y \in \mathbb{R}^{d_y}$, for all $t \geq 0$, $\tau(t) \in [0, \overline{\tau}]$ with $\overline{\tau} > 0$ and an initial condition in C_{in}. The delay $\tau(t)$ is supposed to be a piecewise continuous function. For any $i \in \Xi$, A_i, B_i, and C_i are real and constant matrices of compatible dimensions and σ is a switching signal. We introduce an assumption which pertains to the stabilizability and the detectability of the system (4), but does not imply that all the pairs (A_i, B_i) are stabilizable and all the pairs (A_i, C_i) are detectable.

Assumption 1. There are matrices K_i and L_i for all $i \in \Xi$ and constants $T \geq \overline{\tau}$, $a \in [0, 1)$, $b \geq 0$, $c \in [0, 1)$ and $d \geq 0$ such that the solutions of the system

$$\dot{\alpha}(t) = M_{\sigma(t)}\alpha(t) + \zeta(t) \tag{5}$$