Automatica 93 (2018) 333–342

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive Kalman filter for actuator fault diagnosis☆

Qinghua Zhang

Inria, IFSTTAR, Université de Rennes, Campus de Beaulieu, 35042 Rennes Cedex, France

ARTICLE INFO

Article history:
Received 29 March 2017
Received in revised form 15 November 2017
Accepted 21 February 2018

Keywords:
Adaptive observer
Joint state-parameter estimation
Fault diagnosis
Kalman filter

ABSTRACT

An adaptive Kalman filter is proposed in this paper for actuator fault diagnosis in discrete time *stochastic* time varying systems. By modeling actuator faults as parameter changes, fault diagnosis is performed through joint state-parameter estimation in the considered stochastic framework. Under the classical uniform complete observability–controllability conditions and a persistent excitation condition, the exponential stability of the proposed adaptive Kalman filter is rigorously analyzed. In addition to the minimum variance property of the combined state and parameter estimation errors, it is shown that the parameter estimation within the proposed adaptive Kalman filter is equivalent to the recursive least squares algorithm formulated for a fictive regression problem. Numerical examples are presented to illustrate the performance of the proposed algorithm.

1. Introduction

In order to improve the performance and the reliability of industrial systems, and to satisfy safety and environmental requirements, researches and developments in the field of fault detection and isolation (FDI) have been continuously progressing during the last decades (Hwang, Kim, Kim, & Seah, 2010). Model-based FDI have been mostly studied for linear time invariant (LTI) systems (Chen & Patton, 1999; Ding, 2008; Gertler, 1998; Isermann, 2005; Patton, Frank, & Clarke, 2000), whereas nonlinear systems have been studied to a lesser extent and limited to some particular classes of systems (Berdjag, Christophe, Cocquempot, & Jiang, 2006; De Persis & Isidori, 2001; Xu & Zhang, 2004). This paper is focused on actuator fault diagnosis for linear time-varying (LTV) systems, *including the particular case* of linear parameter varying (LPV) systems. The problem of fault diagnosis for a large class of nonlinear systems can be addressed through LTV/LPV reformulation and approximations (Lopes dos Santos et al., 2011; Tóth, Willems, Heuberger, & Van den Hof, 2011). It is thus an important advance in FDI by moving from LTI to LTV/LPV systems.

In this paper, actuator faults are modeled as parameter changes, and their diagnosis is achieved through joint estimation of states and parameters of the considered LTV/LPV systems. Usually the problem of joint state-parameter estimation is solved by recursive algorithms known as *adaptive observers*, which are most often studied in *deterministic* frameworks for *continuous* time systems (Alma & Darouach, 2014; Besançon, De Leon-Morales, & Huerta-Guevara, 2006; Farza et al., 2014; Farza, M'Saada, Maatouga, & Kamounb, 2009; Liu, 2009; Marino & Tomei, 1995; Tyukin, Steur, Nijmeijer, & van Leeuwen, 2013; Zhang, 2002). Discrete time systems have been considered in Guyader and Zhang (2003), Ţiclea and Besançon (2012) and Ţiclea and Besançon (2016), also in *deterministic* frameworks. In order to take into account *random uncertainties* with a numerically efficient algorithm, this paper considers *stochastic* systems in discrete time, with an *adaptive Kalman filter*, which is structurally inspired by adaptive observers (Guyader & Zhang, 2003; Zhang, 2002), but with well-established stochastic properties.

The main contribution of this paper is an adaptive Kalman filter for discrete time LTV/LPV system joint state-parameter estimation in a *stochastic* framework, *with rigorously proved stability and minimum variance properties*. Its behavior regarding parameter estimation, directly related to actuator fault diagnosis, is well analyzed through its relationship with the recursive least squares (RLS) algorithm.

The recent paper (Ţiclea & Besançon, 2016) addresses the same joint state-parameter estimation, but in a deterministic framework, ignoring random uncertainties. The adaptive observer designed by these authors consists of two interconnected Kalman-like observers, as a natural choice in the considered deterministic framework. In contrast, the adaptive Kalman filter proposed in the present paper involves two interconnected parts, one based on the classical Kalman filter for state estimation, and the other on the RLS algorithm for parameter estimation, resulting from an optimal design in the considered stochastic framework.

☆ The material in this paper was partially presented at the 20th World Congress of the International Federation of Automatic Control, July 9–14, 2017, Toulouse, France. This paper was recommended for publication in revised form by Associate Editor Erik Weyer under the direction of Editor Torsten Söderström.

E-mail address: qinghua.zhang@inria.fr.

https://doi.org/10.1016/j.automatica.2018.03.075

Different adaptive Kalman filters have been studied in the literature for state estimation based on inaccurate state-space models. Most of these algorithms address the problem of unknown (or partly known) state noise covariance matrix or output noise covariance matrix (Brown & Rutan, 1985; Mehra, 1970), whereas the case of incorrect state dynamics model is treated as incorrect state covariance matrix. In contrast, *in the present paper*, the new adaptive Kalman filter is designed for actuator fault diagnosis, by jointly estimating states and parameter changes caused by actuator faults.

The adaptive Kalman filter presented in this paper has also been motivated by hybrid system fault diagnosis. In Zhang (2015) an Adaptive Interacting Multiple Model (AdIMM) estimator has been designed for hybrid system actuator fault diagnosis based on the adaptive Kalman filter and the classical IMM estimator, but *without* theoretical analysis of the adaptive Kalman filter. The results of the present paper fill this missing analysis. Actuator fault diagnosis has also been addressed in Zhang and Basseville (2014) with statistical tests in a two-stage solution, which is not suitable for an incorporation in the AdIMM estimator for hybrid systems.

Preliminary results of this study have been presented in the conference paper (Zhang, 2017). The present manuscript contains enriched details, notably the new result in Section 6 about the equivalence between the parameter estimation within the proposed adaptive Kalman filter and the classical RLS algorithm formulated for a fictive regression problem. More numerical examples are also presented to better illustrate the proposed algorithm.

This paper is organized as follows. The considered problem is formulated in Section 2. The proposed adaptive Kalman filter algorithm is presented in Section 3. The stability of the algorithm is analyzed in Section 4. The minimum variance property of the algorithm is analyzed in Section 5. The relationship with the RLS algorithm is presented in Section 6. Numerical examples are presented in Section 7. Concluding remarks are made in Section 8.

2. Problem statement

The discrete time LTV system subject to actuator faults considered in this paper is generally in the form of[1]

$$x(k) = A(k)x(k-1) + B(k)u(k) + \Phi(k)\theta + w(k) \tag{1a}$$

$$y(k) = C(k)x(k) + v(k), \tag{1b}$$

where $k = 0, 1, 2, \dots$ is the discrete time instant index, $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^l$ the input, $y(k) \in \mathbb{R}^m$ the output, $A(k), B(k), C(k)$ are time-varying matrices of appropriate sizes characterizing the nominal state-space model, $w(k) \in \mathbb{R}^n$, $v(k) \in \mathbb{R}^m$ are mutually independent centered white Gaussian noises of covariance matrices $Q(k) \in \mathbb{R}^{n\times n}$ and $R(k) \in \mathbb{R}^{m\times m}$, and the term $\Phi(k)\theta$ represents actuator faults with a known matrix sequence $\Phi(k) \in \mathbb{R}^{n\times p}$ and a constant (or piecewise constant with rare jumps) parameter vector $\theta \in \mathbb{R}^p$.

A typical example of actuator faults represented by the term $\Phi(k)\theta$ is actuator gain losses. When affected by such faults, the nominal control term $B(k)u(k)$ becomes

$$B(k)(I_l - \mathrm{diag}(\theta))u(k) = B(k)u(k) - B(k)\mathrm{diag}(u(k))\theta$$

where I_l is the $l \times l$ identity matrix, the diagonal matrix $\mathrm{diag}(\theta)$ contains gain loss coefficients within the interval $[0, 1]$, and $\Phi(k) \in \mathbb{R}^{n\times l}$ $(p=l)$ is, in this particular case,

$$\Phi(k) = -B(k)\mathrm{diag}(u(k)). \tag{2}$$

[1] There exists a "forward" variant form of the state-space model, typically with $x(k+1) = A(k)x(k) + B(k)u(k) + w(k)$. While this difference is important for control problems, it is not essential for estimation problems, like the one considered in this paper. The form chosen in this paper corresponds to the convention that data are collected at $k = 1, 2, 3, \dots$ and the initial state refers to $x(0)$.

Though the theoretic analyses in this paper assume a constant parameter vector θ, numerical examples in Section 7 show that rare jumps of the parameter vector (rare occurrences of actuator faults) are well tolerated by the proposed adaptive Kalman filter, at the price of transient errors after each jump.

The problem of actuator fault diagnosis considered in this paper is to characterize actuator parameter changes from the input–output data sequences $u(k)$, $y(k)$, and the matrices $A(k)$, $B(k)$, $C(k)$, $Q(k)$, $R(k)$, $\Phi(k)$.

This characterization of actuator parameter changes will be based on a joint estimation algorithm of states and parameters. In the fault diagnosis literature, diagnosis procedures typically include residual generation and residual evaluation (Basseville & Nikiforov, 1993). In this paper, the difference between the nominal value of the parameter vector θ and its recursively computed estimate can be viewed as a residual vector, and its evaluation can be simply based on some thresholds or on more sophisticated decision mechanisms. In this sense, this paper is focused on residual generation only.

An apparently straightforward solution for the joint estimation of $x(k)$ and θ is to apply the Kalman filter to the augmented system

$$\begin{bmatrix} x(k) \\ \theta(k) \end{bmatrix} = \begin{bmatrix} A(k) & \Phi(k) \\ 0 & I_p \end{bmatrix} \begin{bmatrix} x(k-1) \\ \theta(k-1) \end{bmatrix} + \begin{bmatrix} B(k) \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} w(k) \\ 0 \end{bmatrix}$$

$$y(k) = \begin{bmatrix} C(k) & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ \theta(k) \end{bmatrix} + v(k).$$

However, to ensure the stability of the Kalman filter, this *augmented* system should be uniformly completely observable and uniformly completely controllable regarding the state noise (Jazwinski, 1970; Kalman, 1963). Notice that, even in the case of time *invariant* matrices A and C, the augmented system is time varying because of $\Phi(k)$, which is typically time varying. The uniform complete observability of an LTV system is defined as the uniform positive definiteness of its observability Gramian (Jazwinski, 1970; Kalman, 1963). In practice, it is not natural to directly assume properties (observability and controllability) of the *augmented system* (a more exaggerated way would be *directly assuming* the stability of its Kalman filter, or anything else that should be proved!).

In contrast, *in the present paper*, the classical uniform complete observability and uniform complete controllability are assumed for the *original* system (1), in terms of the Gramian matrices defined for the $[A(k), C(k)]$ pair and the $[A(k), Q^{\frac{1}{2}}(k)]$ pair. These conditions, together with a persistent excitation condition (see Assumption 3 formulated later), ensure the stability of the adaptive Kalman filter presented in this paper.

3. The adaptive Kalman filter

In the adaptive Kalman filter, the state estimate $\hat{x}(k|k) \in \mathbb{R}^n$ and the parameter estimate $\hat{\theta}(k) \in \mathbb{R}^p$ are recursively updated at every time instant k. This algorithm involves also a few other recursively updated auxiliary variables: $P(k|k) \in \mathbb{R}^{n\times n}$, $\Upsilon(k) \in \mathbb{R}^{n\times p}$, $S(k) \in \mathbb{R}^{p\times p}$ and a forgetting factor $\lambda \in (0, 1)$.

At the initial time instant $k = 0$, the initial state $x(0)$ is assumed to be a Gaussian random vector

$$x(0) \sim \mathcal{N}(x_0, P_0). \tag{3}$$

Let $\theta_0 \in \mathbb{R}^p$ be the initial guess of θ, $\lambda \in (0, 1)$ be a chosen forgetting factor, and ω be a chosen positive value for initializing $S(k)$, then the adaptive Kalman filter consists of the initialization step and the recursion steps described below. Each part of this algorithm separated by horizontal lines will be commented after the algorithm description.