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Barrier function-based adaptive sliding mode control☆

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ABSTRACT

In this paper, a new barrier function-based adaptive strategy is proposed for first order sliding mode controller. This strategy is applied to a class of first order disturbed systems whose disturbance is bounded with unknown boundary. The proposed barrier strategy can ensure the convergence of the output variable and maintain it in a predefined neighborhood of zero independent of the upper bound of the disturbance, without overestimating the control gain.

1. Introduction

For systems with matching disturbances, the sliding mode control has proven its high efficiency (Utkin, 1992). Indeed, it provides a closed-loop insensitivity to these disturbances and guarantees the finite-time convergence. However, the implementation of the first order sliding mode controllers (FOSMCs) requires the knowledge of the upper bound of disturbances. In practice, this bound is not constant and, moreover, frequently it is unknown. This means that the gains of the FOSMCs are overestimated. This is a main obstacle in the FOSMCs implementation growing the undesired chattering effect (Boiko, 2008).

Recently, two different strategies to create adaptive sliding mode (SM) controllers have been considered in the case where the upper bound of the disturbance exists but it is unknown.

The first strategy of adaptation consists in increasing the gain until the moment when the SM is reached, and then the gain is fixed at this value, ensuring an ideal SM for some interval of time. When the disturbance grows, the SM can be lost, therefore the gain increases to reach it again (Incremona, Cucuzzella, & Ferrara, 2016; Negrete-Chvez & Moreno, 2016). However, the FOSMC gain in this strategy is overestimated and one cannot be sure that the SM will not be lost in the future. To overcome this problem, an approach based on increasing and decreasing the gain has been developed (Bartolini, Levant, Plestan, Taleb, & Punta, 2012; Incremona et al., 2016; Plestan, Shtessel, Bregeault, & Poznyak, 2010; Shtessel, Taleb, & Plestan, 2012). This approach ensures the finite-time convergence of the sliding variable to some neighborhood of zero without big overestimation of the gain. The main drawback of this approach is that the size of the above mentioned neighborhood and the time of convergence depend on the unknown upper bound of disturbance, i.e. they are unknown a priori and one can never be sure that SM will never be lost for bigger values of time.

The second strategy of adaptation is based on the usage of the equivalent control value as an estimation of the disturbance (Bartolini, Ferrara, Pisano, & Usai, 1998; Edwards & Shtessel, 2016; Oliveira, Cunha, & Hsu, 2016; Utkin & Poznyak, 2013). In Oliveira et al. (2016) a model based approach is presented. To realize this strategy a low-pass filtered approximation of the equivalent control were proposed. However, during the realization, the filter constant should be chosen much less than the inverse of the upper bound of the first derivative of disturbance.

The aim of this paper is to propose an adaptive strategy that can achieve the convergence of the output variable to a predefined neighborhood of zero, with a control gain that is not overestimated, and without using any information about the upper bound of the disturbance, nor the use of the low pass filter.

This paper proposes the use of Barrier Functions (BFs) as an adaptive strategy for FOSMC in order to reach above mentioned goal. In this current paper, two different classes of BFs are used: the positive semi-definite BF and a positive definite BF.

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The main advantages of the proposed barrier adaptive SM control are:

- The output variable converges in a finite time to a *predefined neighborhood* of zero, independently of the bound of the disturbance, and cannot exceed it.
- The gain provided by the proposed strategy is *not overestimated*, as it can only achieve the convergence of the output variable to a predefined neighborhood of zero.
- The proposed strategy theoretically *does not require neither the bounds of the disturbance nor the use of the low-pass filter.*

This paper is organized as follows. In Section 2 the problem formulation is given. Section 3 presents the barrier adaptive FOSMC algorithm. Finally, some conclusions are drawn in Section 4.

2. Problem formulation

Consider the first order system

$$\dot{s}(t) = u(t) + \delta(t), \qquad (1)$$

where $s(t) \in \mathbb{R}$ is the output variable, $u(t)$ is the FOSMC and $\delta(t)$ is a disturbance. Here $\delta(t)$ is bounded function with unknown bound, i.e. $|\delta(t)| \leq \delta_{max}$. The bound $\delta_{max} > 0$ exists but is not known.

In this context, the gain of the FOSMC is to be adapted in accordance with the adaptive strategy defined later. The idea behind the proposed adaptive strategy is to first increase the adaptive gain until the output variable reaches a small neighborhood of zero $\frac{\varepsilon}{2}$ at time $\bar{t}$ by using a constant derivative gain as in Plestan et al. (2010). Secondly, for $t > \bar{t}$, the adaptive gain switches to a BF that can maintain the output variable in the predefined neighborhood of zero $|s(t)| < \varepsilon$.

2.1. Preliminaries

2.1.1. Barrier functions (BFs)

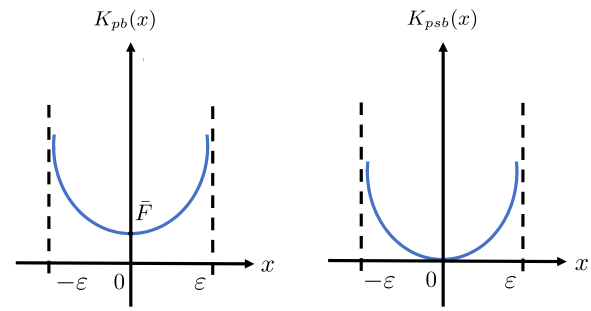
Definition 1. Let us suppose that some $\varepsilon > 0$ is given and fixed, the BF can be defined as an even continuous function $K_b : x \in]-\varepsilon, \varepsilon[\rightarrow K_b(x) \in [b, \infty[$ strictly increasing on $[0, \varepsilon[$.

- $\lim_{|x| \to \varepsilon} K_b(x) = +\infty$.
- $K_b(x)$ has a unique minimum at zero and $K_b(0) = b \geq 0$.

In this paper, the following two different classes of BFs are considered;

- Positive definite BFs (PBFs): $K_{pb}(x) = \frac{\varepsilon \bar{F}}{\varepsilon - |x|}$, i.e. $K_{pb}(0) = \bar{F} > 0$.
- Positive Semi-definite BFs (PSBFs): $K_{psb}(x) = \frac{|x|}{\varepsilon - |x|}$, i.e. $K_{psb}(0) = 0$.

The PBF $K_{pb}(x)$ and the PSBF $K_{psb}(x)$ are illustrated in Fig. 1.



(a) $K_{pb}(x)$. (b) $K_{psb}(x)$.

Fig. 1. Schematic illustrations of $K_{pb}(x)$ and $K_{psb}(x)$.

3. Barrier adaptive FOSMC algorithm

The following theorem is true for both possible FOSMC gains design: using $K_B(s(t)) = K_{pb}(s(t))$ and $K_B(s(t)) = K_{psb}(s(t))$.

Theorem 2. *Consider system (1) with bounded disturbance $\delta(t)$ with the controller*

$$u(t) = -K(t, s(t))sign(s(t)), \qquad (2)$$

and with the adaptive control gain $K(t, s)$

$$K(t, s(t)) = \begin{cases} K_a(t), \dot{K}_a(t) = \bar{K}|s(t)|, & if\ 0 < t \leq \bar{t} \\ K_B(s(t)), & if\ t > \bar{t} \end{cases} \qquad (3)$$

where $\bar{K}$ to be arbitrary positive constant.

Then for any $s(0)$ and $\varepsilon > 0$, there exists $\bar{t}$ the smallest root of equation $|s(t)| \leq \frac{\varepsilon}{2}$ such that for all $t \geq \bar{t}$, the inequality $|s(t)| < \varepsilon$ holds.

The proof of Theorem 2 is given in the Appendix.

Remark 3. Note that this strategy allows the adaptive gain to increase and decrease based on the current value of the output variable. When the output variable is going to zero, the adaptive gain decreases till the value which allows to compensate the disturbance.

On the other hand, when the disturbance grows and the control gain is less than the absolute value of disturbance, the output variable grows and the control gain can grow if it is necessary till the level ensuring that the system solution will never leave the ε vicinity of zero.

Remark 4. Theoretically, the a priori knowledge of actuator capacity P is not required, but it should be supposed that the actuator is capable to compensate the disturbance. However, in practice, an actuator is used and its capacity P is known. In this case for discrete implementation of the proposed algorithm, the sampling step τ should be chosen as $\tau \ll \varepsilon/P$. Otherwise, the attractive feature of the BF will be lost, and the output variable will leave the predefined neighborhood of zero.

The behavior of each barrier function PBF and PSBF, and the achievement of real or ideal SM in finite time, together with the continuity or discontinuity of the control signal are discussed in Sections 3.1 and 3.2.

3.1. Adaptation with PBF

Consider the adaptation with PBF. In this case, $K_{pb}(s(t))$ has a lower bound $\bar{F}$ when $s(t) = 0$. Therefore, when $|\delta(t)| < \bar{F}$ the adaptive gain is overestimated. In this case, this strategy provides an ideal SM. In order to attenuate this overestimation, $\bar{F}$ can be chosen small enough. The usage of PBF when the bound of the disturbance is less than $\bar{F}$ will provide a discontinuous control signal leading to the chattering whose amplitude is proportional to the choice of $\bar{F}$.

3.2. Adaptation with PSBF

Consider now the adaptation with PSBF. In this case, $K_{psb}(s(t))$ tends to zero when $s(t) \rightarrow 0$. Hence, $K_{psb}(s(t))$ has the same behavior as $\frac{|s(t)|}{\varepsilon}$ in the neighborhood of zero, i.e. $\frac{|s(t)|}{\varepsilon} \ll 1 \rightarrow K_{psb}(s(t)) = \frac{\frac{|s(t)|}{\varepsilon}}{\varepsilon - |s(t)|} \approx \frac{|s(t)|}{\varepsilon}$.

This means that if $\delta(t)$ and $s(t)$ tend monotonically to zero, consequently the adaptive gain $K_{psb}(s(t))$ will go to zero. The discontinuity of the control signal can appear only once at time $\bar{t}$,