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Leader-following attitude consensus of multiple rigid body systems subject to jointly connected switching networks☆

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ABSTRACT

The leader-following attitude consensus problem of multiple rigid body systems has been studied by the distributed observer approach. The key assumption in the existing results is that the communication network among the rigid body systems is static and connected. Nevertheless, this assumption is undesirable since, typically, the communication network is time-varying and disconnected from time to time due to changes of the environment or failures of some subsystems. In this paper, we will further study the leader-following attitude consensus problem of multiple rigid body systems subject to a jointly connected switching communication network. This new problem is more challenging than the existing one since a jointly connected switching communication network can be disconnected at every time instant. To overcome the difficulty, we first show that the distributed observer for a nonlinear target system subject to a jointly connected switching communication network exists. Then, we further synthesize a distributed control law utilizing this distributed observer for the multiple rigid body systems. Finally, we show that this distributed control law solves our problem through the argument of the certainty equivalence principle.

1. Introduction

The attitude control of rigid body systems has received constant attention not only because many space missions and robot applications demand precise attitude control, but also because the problem poses some specific challenges to control theory and technology. The problem has been studied under various scenarios with a variety of techniques in, say, Chen & Huang (2009, 2015), Luo, Chu, & Ling (2005), Sidi (1997), Tayebi (2008), and Yuan (1988). Recently, as more and more space missions are performed through coordinated operations of multiple spacecraft systems, the attitude consensus problem of multiple rigid body systems is getting more and more attentions from the control community.

There are two types of attitude consensus problems for multiple rigid body systems: the leaderless attitude consensus problem and the leader-following attitude consensus problem. The leaderless attitude consensus problem is also called the attitude synchronization problem. It aims to synchronize the attitudes of all rigid body systems to a common trajectory, which is determined by the initial states of all systems (Abdessameud & Tayebi, 2009; Lawton & Beard, 2002). On the other hand, the leader-following attitude consensus problem aims to drive the attitudes of all rigid body systems to a desirable trajectory generated by a target system (Bai, Arcak, & Wen, 2008; Cai & Huang, 2014, 2016; Ren, 2007). In particular, Cai & Huang (2014, 2016) studied the leader-following attitude consensus problem of multiple rigid body systems by employing a so-called distributed observer, which is a dynamic compensator that can provide for each rigid body system the estimates of the angular velocity and the attitude of the target system. The results in Cai and Huang (2014, 2016) were obtained under the condition that the communication network of the multiple rigid body systems is static and connected, which is the least restrictive condition in the existing literature. Nevertheless, in some real applications, the assumption that the communication network is static and connected may be undesirable since, typically, the communication network is time-varying and disconnected from time to time due to changes of the environment, link failures, or network reconfigurations.

In this paper, we will further study the more practical and more desirable scenario where the communication network is a jointly connected switching network (Jadbabaie, Lin, & Morse, 2003). The jointly connected assumption is the mildest assumption among all existing assumptions on the communication network because,

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Nomenclature

$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbf{1}_N$	An N dimensional column vector whose components are all 1
$\otimes$	Kronecker product
$\|\cdot\|$	Euclidean norm of a vector or induced Euclidean norm of a matrix
$\mathrm{col}(\cdot)$	For $X_i \in \mathbb{R}^{n_i \times p}, i = 1, \ldots, m$ $\mathrm{col}(X_1, \ldots, X_m) = \left[X_1^T \ \cdots \ X_m^T\right]^T$
$q(\cdot)$	For $x \in \mathbb{R}^3$, $q(x) = \mathrm{col}\,(x, 0) \in \mathbb{R}^4$
$\mathbb{Q}$	Set of all quaternions $\mathbb{Q} = \{q \mid q = \mathrm{col}(\hat{q}, \bar{q}), \hat{q} \in \mathbb{R}^3, \bar{q} \in \mathbb{R}\}$
$\mathbb{Q}_u$	Set of all unit quaternions $\mathbb{Q}_u = \{q \mid q \in \mathbb{Q},\ \|q\| = 1\}$
q_I	Quaternion identity, $q_I = \mathrm{col}(0, 0, 0, 1) \in \mathbb{Q}_u$
$(\cdot)^*$	Quaternion conjugate, for $q \in \mathbb{Q}$, $q^* = \mathrm{col}(-\hat{q}, \bar{q})$
$(\cdot)^{-1}$	Quaternion inverse, for $q \in \mathbb{Q}_u$, $q^{-1} = q^*$
$\odot$	Quaternion product, for $q_i, q_j \in \mathbb{Q}$ $q_i \odot q_j = \begin{bmatrix} \bar{q}_i\hat{q}_j + \bar{q}_j\hat{q}_i + \hat{q}_i^{\times}\hat{q}_j \\ \bar{q}_i\bar{q}_j - \hat{q}_i^T\hat{q}_j \end{bmatrix}$
$C(\cdot)$	For $q \in \mathbb{Q}$, $C(q) = (\bar{q}^2 - \hat{q}^T\hat{q})I_3 + 2\hat{q}\hat{q}^T - 2\bar{q}\hat{q}^{\times}$ If $q \in \mathbb{Q}_u$, $C(q)$ is the direction cosine matrix
$(\cdot)^{\times}$	For $x = \mathrm{col}(x_1, x_2, x_3) \in \mathbb{R}^3$ $x^{\times} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 3}$
$\mathcal{I}$	Inertial frame
$\mathcal{B}_0$	Body frame of the target system
$\mathcal{B}_i$	Body frame of the ith rigid body
q_0	The attitude of $\mathcal{B}_0$ relative to $\mathcal{I}$
q_i	The attitude of $\mathcal{B}_i$ relative to $\mathcal{I}$
ω_0	The angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$
ω_i	The angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$
J_i	Inertia matrix of the ith rigid body
u_i	Control torque of the ith rigid body
S	System matrix of the angular velocity target system
E	Output matrix of the angular velocity target system
v	State of the angular velocity target system
ϵ_i	Relative attitude between $\mathcal{B}_i$ and $\mathcal{B}_0$ $\epsilon_i = q_0^{-1} \odot q_i$
$\hat{\epsilon}_i$	The vector part of ϵ_i, $\epsilon_i = \mathrm{col}(\hat{\epsilon}_i, \bar{\epsilon}_i)$
$\check{\omega}_i$	Angular velocity of $\mathcal{B}_i$ relative to $\mathcal{B}_0$ $\check{\omega}_i = \omega_i - C(\epsilon_i)\omega_0$
ξ_i	Estimate of v
ζ_i	Estimate of ω_0
η_i	Estimate of q_0
e_i	Estimate of ϵ_i, $e_i = \eta_i^* \odot q_i$
$\hat{e}_i$	The vector part of e_i, $e_i = \mathrm{col}(\hat{e}_i, \bar{e}_i)$
$\tilde{\omega}_i$	Estimate of $\check{\omega}_i$, $\tilde{\omega}_i = \omega_i - C(e_i)\zeta_i$

under this assumption, the network can be disconnected at every time instant, and it includes the static and connected network as a special case. Nevertheless, the jointly connected switching network together with the nonlinearity of the target system poses two specific challenges that cannot be handled by the approach in Cai and Huang (2014, 2016). First, the establishment of the distributed observer in Cai and Huang (2014, 2016) explicitly relies on a Lyapunov function candidate for the distributed observer which exists only if the network is static and connected. Here, we will connect the stability of our distributed observer to a newly established stability result for a perturbed linear switched system given recently in Liu and Huang (2017). Second, in order to overcome the nonlinearity of the target system, we will make use of a pseudo linear representation of the attitude kinematics of the target system. This new representation allows us to apply the result in Liu and Huang (2017) to our case. In particular, for the special case studied in Cai and Huang (2014, 2016), where the network is static and connected, our new approach will simplify the convergence analysis of the distributed observer in Cai and Huang (2014, 2016).

It is noted that the leaderless consensus problem of rotation groups in $SO(3)$ and $SO(n)$ were studied in Matni and Horowitz (2014) and Tron, Afsari, and Vidal (2012), respectively. Instead of proposing distributed control laws to control the dynamics of each system, distributed algorithms for reaching consensus were proposed in Matni and Horowitz (2014) and Tron et al. (2012) by solving optimization problems.

The remainder of the paper is organized as follows. In Section 2, we give a formulation of our problem and list two assumptions for the solvability of the problem. In Section 3, we focus on establishing the existence of the distributed observer subject to jointly connected switching networks. In Section 4, we further synthesize a distributed control law utilizing this distributed observer, and show that this distributed control law solves our problem through the argument of the certainty equivalence principle. In Section 5, an example is presented to illustrate the effectiveness of our approach. Finally, we conclude this paper in Section 6 with some remarks.

2. Problem formulation and preliminaries

In this paper, we use unit quaternion to represent the attitude of a rigid body with respect to the inertial frame. As in Cai and Huang (2014), we consider a group of N rigid bodies, whose attitude kinematics and dynamics are governed by the following equations:

$$\dot{q}_i = \frac{1}{2} q_i \odot q(\omega_i) \tag{1a}$$

$$J_i\dot{\omega}_i = -\omega_i^{\times} J_i\omega_i + u_i, \quad i = 1, \ldots, N \tag{1b}$$

where $q_i \in \mathbb{Q}_u$ is the unit quaternion representation of the attitude of the frame $\mathcal{B}_i$ relative to the inertial frame $\mathcal{I}$; $\omega_i \in \mathbb{R}^3$ is the angular velocity of the frame $\mathcal{B}_i$ relative to the inertial frame $\mathcal{I}$; $J_i \in \mathbb{R}^{3\times 3}$ is the positive definite inertia matrix and $u_i \in \mathbb{R}^3$ is the control torque of the ith rigid body. Note that ω_i, J_i, and u_i are all expressed in $\mathcal{B}_i$.

Like in Cai and Huang (2016), we assume that the desired angular velocity $\omega_0 \in \mathbb{R}^3$ and attitude $q_0 \in \mathbb{Q}_u$ of the target system's fixed body frame $\mathcal{B}_0$ relative to the inertial frame $\mathcal{I}$ are governed by the following equations:

$$\dot{v} = Sv, \quad \omega_0 = Ev \tag{2a}$$

$$\dot{q}_0 = \frac{1}{2} q_0 \odot q(\omega_0) \tag{2b}$$

where $v \in \mathbb{R}^m$, and $S \in \mathbb{R}^{m\times m}$, $E \in \mathbb{R}^{3\times m}$ are constant matrices.

Also, as in Cai and Huang (2014), we view the system composed of (1) and (2) as a multi-agent system of $(N+1)$ agents with (2) as the leader and the N subsystems of (1) as followers. However, here the communication network of the multi-agent system is described by a switching digraph[1] $\bar{\mathcal{G}}_{\sigma(t)} = \left(\bar{\mathcal{V}}, \bar{\mathcal{E}}_{\sigma(t)}\right)$ with $\sigma(t)$ being a piecewise constant switching signal, $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$, and $\bar{\mathcal{E}}_{\sigma(t)} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ for all $t \geq 0$. Node 0 is associated with the leader system (2) and node i, $i = 1, \ldots, N$, is associated with

[1] See Appendix A for a summary of notation on digraph.