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Global stabilization via sampled-data output feedback for large-scale systems interconnected by inherent nonlinearities[☆]

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ABSTRACT

This paper aims to solve the decentralized sampled-data control problem for a class of large-scale systems interconnected by inherent nonlinearities, which violate the linear growth condition. Under a homogeneous growth condition, a set of decentralized sampled-data output feedback controllers are constructed with scaling gains and a sampling period. By the output feedback domination approach, the scaling gains are firstly correlated and then selected to dominate the nonlinearities. Finally, by developing and applying a useful tool to estimate trajectory growth, global stabilization is achieved for the closed-loop system under an appropriate sampling period.

1. Introduction

Decentralized control for large-scale nonlinear systems has been an active topic in control community, for example, see Liu, Jiang, and Hill (2012), Wang, Wen, and Guo (2016), Ye and Huang (2003) and Yu, Xie, and Wu (2011). Many interesting results have been received for a class of large-scale interconnected uncertain nonlinear systems described by

$$\begin{aligned} \dot{x}_{ij}(t) &= x_{i,j+1}(t) + \phi_{ij}(t, x(t)),\ j = 1, 2, \ldots, n-1, \\ \dot{x}_{in}(t) &= u_i(t) + \phi_{in}(t, x(t)), \\ y_i(t) &= x_{i1}(t), \qquad\qquad i = 1, 2, \ldots, m, \end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$, $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^{nm}$ are system states, $u_i(t) \in \mathbb{R}$ is control input, $y_i(t) \in \mathbb{R}$ is the system output, and $\phi_{ij}(t, x)$ is an uncertain continuous function. When only the outputs y_i's are available, the problem of decentralized control by output feedback for system (1) has been known more challenging even in the case of using continuous-time controllers. The major difficulty is that each subsystem is impacted by its own unmeasurable states as well as the unmeasurable states of other subsystems, while the decentralized control can only utilize its own output (Frye, Qian, & Colgren, 2005; Jiang, 2000; Jiang, Repperger, & Hill, 2001; Labibi, Lohmann, Sedigh, & Maralani, 2002; Zheng, 1989).

Since in practice more and more controllers are being implemented by digital computers, the design and use of sampled-data controllers for system (1) have become imperative and much more challenging. Recently, it has been shown that the emulation method, which discretizes a globally stabilizing continuous-time controller, can guarantee global stability of the closed-loop system with the discretized controller under certain conditions (Khalil, 2004; Nešić, Teel, & Carnevale, 2009). In a recent work (Qian & Du, 2012), a global stabilizer via sampled-data output feedback is constructed for a class of nonlinear systems under a linear growth condition. Nevertheless, all these results only focus on single system. For large-scale systems, although there are many existing continuous-time controllers, discretizing them will only lead to local results due to the phenomenon of finite escape time demonstrated in Zhang, Jia, Qian, and Li (2015, Example (1.1)). For global result, under a linear growth condition the result in Qian and Du (2012) has been extended to the interconnected nonlinear system (1) in Zhang, Qian, and Li (2013). However, all the aforementioned results are achieved under various linear growth conditions which certainly limit the class of nonlinear systems that can be globally stabilized via sampled-data output feedback. The question of how to design decentralized global stabilizers via sampled-data output feedback for interconnected system (1) with *nonlinearly growing* nonlinearities is still open. For example, the

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following system interconnected by inherent nonlinearities

$$\dot{x}_{11} = x_{12} + |x_{11}|, \dot{x}_{12} = u_1 + x_{22}^3, y_1 = x_{11},$$
$$\dot{x}_{21} = x_{22}, \dot{x}_{22} = u_2 + \sin x_{22} + \ln(1 + x_{12}^2), y_2 = x_{21} \quad (2)$$

cannot be solved by the method in Zhang et al. (2013) since the nonlinearity x_{22}^3 does not satisfy the linear growth condition.

There are three major obstacles on the path to design sampled-data output feedback controllers which globally stabilize (2). First, due to the existence of the higher-order nonlinear terms, the approach in Zhang et al. (2013) cannot be applied directly. As a matter of fact, the emulation method highly relies on the estimation of the trajectory growth of a closed-loop system under a sampled-data controller. The inherent nonlinearities such as x_{22}^3 make it difficult to estimate the trajectory growth of (2) under sampled-data controllers. To overcome this obstacle, we adopt the idea of homogeneous domination (Polendo & Qian, 2007) to categorize the inherent nonlinearities under the perspective of homogeneity. Under this framework, we also develop a new tool to estimate the trajectory growth of the interconnected systems with inherent nonlinearities. Second, the nonlinearities of each subsystem grow at different rates and hence a unified scaling gain utilized in the traditional homogeneous domination method is not efficient. To tackle this issue, in this paper we propose to use multiple scaling gains in observers and controllers and link them using the intrinsic homogeneous relations among the subsystems. The third obstacle emerges from the stability analysis of the closed-loop system since each subsystem has a different convergence rate due to the different homogeneous degrees. To overcome this obstacle, we construct a homogeneous-like Lyapunov function, instead of a quadric function, to characterize the different convergence rates of the subsystems.

With the proposed new estimation tool, multiple-gain domination methodology, and stability analysis procedure, a set of decentralized controllers consisting of sampled-data observers and control laws are designed for system (1) in this paper. With appropriate scaling gains and a carefully selected sampling period, the closed-loop system is rendered globally asymptotically stable even in the presence of inherent nonlinearities which violate the linear growth condition. Despite the inherent nonlinearities, the proposed decentralized controllers are linear in nature and easy to be implemented.

2. Problem statement

This paper aims to design sampled-data output feedback controllers to globally stabilize the large-scale uncertain nonlinear system (1). As shown in Qian and Du (2012), under the linear construction, a continuous-time output feedback controller can be digitized to an equivalent sampled-data controller without loss of accuracy. In addition, it is very easy to implement linear sample-data controllers by computers. To globally stabilize the large-scale system (1), we also focus on constructing linear sampled-data output feedback controllers in this paper. More specifically, we are interested in solving the following problem:

Global Stabilization of (1) by Linear Sampled-Data Output Feedback: Construct linear sampled-data output feedback controllers of the form

$$\xi_i(t_{k+1}) = M_i\xi_i(t_k) + N_i y_i(t_k), \quad t_k = kT, \quad (3)$$
$$u_i(t) = -\bar{K}_i\xi_i(t_k), \forall t \in [t_k, t_{k+1}), \ k = 0, 1, \ldots \quad (4)$$

with matrices $M_i \in \mathbb{R}^{n\times n}$, $N_i \in \mathbb{R}^n$, and $\bar{K}_i \in \mathbb{R}^{1\times n}$ for $i = 1, \ldots, m$, such that the closed-loop system (1), (3) and (4) is globally asymptotically stable for an appropriate sampling period T.

Assumption 2.1. For $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, there exist positive constants r_i's and c_i's such that

$$|\phi_{ij}(t, x)| \le c_i \sum_{l=1}^{m}\sum_{k=1}^{j} |x_{lk}|^{\frac{r_i}{r_l}}, \forall t \in \mathbb{R}, x \in \mathbb{R}^{nm}. \quad (5)$$

Remark 2.1. Assumption 2.1 presents a nonlinear growth condition, which covers a large-scale uncertain nonlinear system interconnected in particular by inherent nonlinearities. When $r_i = r_j, \forall i, j$, this growth condition degenerates to the linear growth condition under which a linear sampled-data output feedback controller has been designed to globally stabilize system (1) in Zhang et al. (2013), including two inverted pendulums connected by a spring considered in Spooner and Passino (1999). When $r_i \neq r_j, \forall i, j$, Assumption 2.1 can cover more general nonlinearities such as those in system (2) (see discussion in Example 3.1).

3. Main results

To solve the problem, first we recall three useful lemmas.

Lemma 3.1 (Hardy, Littlewood, & Polya, 1952)**.** *For any $a_i \in \mathbb{R}$, $i = 1, \ldots, n$, and real number $p \ge 1$,*

(i) $(|a_1| + \cdots + |a_n|)^p \le n^{p-1}(|a_1|^p + \cdots + |a_n|^p)$,

(ii) $(|a_1| + \cdots + |a_n|)^{1/p} \le |a_1|^{1/p} + \cdots + |a_n|^{1/p}$.

Lemma 3.2 (Hardy et al., 1952)**.** *Let $f(\cdot)$ and $g(\cdot)$ be real-valued functions continuous over the interval $[a, b]$ with $a \le b$. For a constant $p \ge 1$, the following holds:*

$$\left|\int_a^b f(s)g(s)ds\right|^p \le \left(\int_a^b |g(s)|^{\frac{p}{p-1}}ds\right)^{p-1}\int_a^b |f(s)|^p ds.$$

Lemma 3.3 ((Hardy et al., 1952) *Young Inequality*)**.** *For positive constants c and d, the following inequality holds:*

$$|x|^c|y|^d \le \frac{c}{c+d}|x|^{c+d} + \frac{d}{c+d}|y|^{c+d}.$$

With the help of above three lemmas, the following lemma provides a tool to estimate trajectory growth of an interconnected hybrid system.

Lemma 3.4. *Consider the large-scale nonlinear system*

$$\dot{z}_i(t) = F_i z_i(t) + G_i z_i(t_k) + J_i(t, z), \ t \in [t_k, t_{k+1}) \quad (6)$$

for $i = 1, 2, \ldots, m$, where $z_i \in \mathbb{R}^n$ is the state, $F_i, G_i \in \mathbb{R}^{n\times n}$ are constant matrices, and the terms $J_i(\cdot)$ satisfy $\|J_i(\cdot)\| \le \bar{c}_i \sum_{l=1}^m \|z_l(t)\|^{r_i/r_l}$, for positive constants $\bar{c}_i$'s and r_i's. Then, there is a class $\mathcal{K}$ function $\delta(\cdot)$ defined on $[0, T^*)$ for a constant $T^* > 0$ such that the following holds:

$$\sum_{i=1}^{m} \|z_i(t) - z_i(t_k)\|^{r_s/r_i} \le \delta(T)\sum_{l=1}^{m}\|z_l(t)\|^{r_s/r_l}, \quad (7)$$

where $r_s := \max_{1\le i\le m}\{r_i\}$.

Proof. Integrating the i-subsystem over $[t_k, t]$ results in

$$\|z_i(t) - z_i(t_k)\| \le \|F_i\| \int_{t_k}^t \|z_i(\theta)\|d\theta$$
$$+ \|G_i\| \int_{t_k}^t \|z_i(t_k)\|d\theta + \bar{c}_i \sum_{l=1}^{m}\int_{t_k}^t \|z_l(\theta)\|^{\frac{r_s}{r_l}}d\theta$$
$$\le (\|F_i\| + \|G_i\|)\int_{t_k}^t \Big(\|z_i(\theta) - z_i(t_k)\| + \|z_i(t_k)\|\Big)d\theta$$

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