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Brief paper

Receding horizon consensus of general linear multi-agent systems with input constraints: An inverse optimality approach☆

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ABSTRACT

This paper investigates the optimal consensus problem for general linear MASs (of semi-stable and unstable dynamics) subject to control input constraints. The optimal consensus protocols are first designed by inverse optimality approach, based on which the centralized receding horizon control (RHC)-based consensus strategies are designed and the feasibility and consensus properties of the closed-loop systems are analyzed. Utilizing the centralized one, distributed RHC-based consensus strategies are developed. We show that (1) the optimal performance indices under the inverse optimal consensus protocols are coupled with the network topologies and the system matrices of subsystems; (2) the unstable modes of subsystems impose more stringent requirements for the parameter design; (3) the designed RHC-based consensus strategies can make the control input constraints fulfilled and ensure *convergent consensus* and consensus for MASs with semi-stable and unstable subsystems, respectively.

1. Introduction

The consensus problem is one of the most important issues in multi-agent systems (MASs). It finds many applications in multi-robotic systems, sensor networks, and power grids, and is also essential to solve some other problems such as formation control, swarm, and distributed estimation problems. Many celebrated results on consensus have been developed, for example, Olfati-Saber and Murray (2004) and Ren and Beard (2005). Even though much progress has been made in MASs, many practical issues in consensus protocol design are still left to be explored.

The optimality is a practical requirement in many control systems, and it is also a desired property for consensus protocol design in MASs. For instance, a wireless sensor network may be expected to reach consensus in state estimates using smallest energy as each sensor node has limited battery power. In addition, the optimal consensus protocol may provide some satisfactory control performance as in LQR. This motivates the study of optimal consensus problem in e.g., Borrelli and Keviczky (2008), Cao and Ren (2010) and Hengster-Movric and Lewis (2014). Another frequently encountered issue would be the control input constraints in MASs. For example, in a multi-robot system, the control inputs for motors in each robot are not allowed to be too large in order not to ruin the motors, or the motors may not provide enough power to generate very large control inputs. The typical results for consensus with input constraints can be found in Nedic, Ozdaglar, and Parrilo (2010) and Lin and Ren (2014).

It is well known that the receding horizon control (RHC) strategy, also known as model predictive control is capable of handling system constraints while preserving (sub-)optimal control performance, and this motivates us to study the constrained consensus problem in an RHC-based framework. In this paper, we consider two classes of discrete-time linear MASs, i.e., MASs with semi-stable and unstable subsystems (i.e., not semi-stable). For both classes of MASs, we first investigate the inverse optimal consensus problem and design optimal consensus protocols, and then study the RHC-based consensus problems and investigate the feasibility issue and analyze the achieved consensus property.

In the literature of RHC strategy for MASs, most of the results are focused on cooperative stabilization problems by either assuming fully-connected networks or avoiding analyzing detailed network affects (e.g., Franco, Magni, Parisini, Polycarpou, & Raimondo, 2008; Li & Shi, 2014; Müller, Reble, & Allgöwer, 2012),

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with a few exceptions (Ferrari-Trecate, Galbusera, Marciandi, & Scattolini, 2009; Li, Shi, & Yan, 2016; Li & Yan, 2015; Zhan & Li, 2013; Zhang, Cheng, & Chen, 2015). In Ferrari-Trecate et al. (2009), the RHC-based consensus strategies are proposed for MASs with integrator and double-integrator dynamics. In Zhan and Li (2013), the consensus problem for MASs with integrator is solved by using unconstrained RHC, requiring multiple-time information exchange. In Zhang et al. (2015), the RHC-based consensus problem is studied for MASs with double-integrator and input constraints. However, this method may not be directly applicable for MASs with higher order dynamics. The RHC-based consensus problem is investigated for MASs with general linear dynamics in Li and Yan (2015), but the input constraints are not considered.

In this paper, we propose a solution to the RHC-based consensus problem for general linear MASs with input constraints. The main contributions of this paper are as follows:

- The global optimal consensus protocols and the conditions for designing such protocols are proposed for MASs with semi-stable and unstable subsystems, based on which novel centralized RHC-based consensus strategies that can fulfill control input constraints are developed. The conditions for decomposition of cost functions and constraints are provided, and the distributed RHC-based consensus strategies are designed for MASs with constraints.
- The feasibility and consensus properties are analyzed for both classes of MASs. We prove that, the designed RHC algorithm is feasible and the closed-loop system can reach consensus. In particular, for the MASs with semi-stable subsystems, the *convergent consensus* can be guaranteed.

Notation: The superscripts "T", "−1" and "#" are denoted by the matrix transposition, inverse and group inverse, respectively. $\mathbb{R}$ ($\mathbb{R}_{\geqslant 0}$) and $\mathbb{Z}$ ($\mathbb{Z}_{+}$) represent the real numbers (nonnegative real numbers) and integers (nonnegative integers), respectively. For a matrix P, $P > 0$ ($P \geqslant 0$) means it is positive-definite (semi-positive definite). For a vector $x \in \mathbb{R}^n$, its Euclidean norm and P-weighted norm are denoted by $\|x\|$ and $\|x\|_P \triangleq \sqrt{x^{\mathrm{T}}Px}$, respectively, where $P \geqslant 0$. The distance between x and a set $\mathcal{O} \subseteq \mathbb{R}^n$ is denoted by $|x|_{\mathcal{O}} = \inf_{y\in\mathcal{O}}\|x-y\|$. Given a matrix P, we use $\lambda(P)$, spec(P), $\sigma_{\min}(P)$ and $\sigma_{\max}(P)$ to represent its eigenvalue, spectrum radius, minimum and maximum nonzero spectrum, respectively. For a matrix A, its range and null space are denoted by range(A) and Null(A), respectively. We write the column operation $[x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_n^{\mathrm{T}}]^{\mathrm{T}}$ as col$(x_1, x_2, \ldots, x_n)$. Given two sets $A \subseteq B \subseteq \mathbb{R}^n$, the difference between them is defined by $A \setminus B \triangleq \{x | x \in A, x \notin B\}$. $\otimes$ denotes the Kronecker product operation.

2. Problem formulation

Consider an MAS with each agent i

$$x_{k+1}^i = Ax_k^i + Bu_k^i,\ i = 1, \ldots, M, \tag{1}$$

where $x_k^i \in \mathbb{R}^n$ is the state, $u_k^i \in \mathbb{R}^m$ is the control input. The control input needs to satisfy the constraint $u_k^i \in \mathcal{U}_i$, where $\mathcal{U}_i$ are compact sets, and contain the origin as their interior points. Each agent i can communicate with some neighboring agents via the communication network, which is characterized by a graph $\mathcal{G}$ with a triple $\{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{1, \ldots, N\}$ represents the collection of N vertices (nodes), $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of arcs or edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} \geqslant 0$ is the weighted adjacency matrix of the graph $\mathcal{G}$.

It is assumed that $\mathcal{G}$ contains no-self loop, and its Laplacian matrix is denoted by $\mathcal{L}$. In $\mathcal{G}$, the neighboring set for agent i is denoted by $\mathcal{N}_i$. For more graph notations, definitions and properties, the reader is referred to Ren and Beard (2005) and You and Xie (2011).

The overall augmented system can be written as

$$X_{k+1} = (I_M \otimes A)X_k + (I_M \otimes B)U_k, \tag{2}$$

where $X_k = \mathrm{col}(x_k^1, \ldots, x_k^M)$, and $U_k = \mathrm{col}(u_k^1, \ldots, u_k^M)$. The system constraint becomes $U_k \in \mathcal{U}$, where $\mathcal{U} = \mathcal{U}^1 \times \cdots \times \mathcal{U}^M$.

Definition 1. For the MAS of dynamics in (1) over a graph $\mathcal{G}$, with certain control input u_k^i to close the loop, it is said to reach consensus, if $\lim_{k\to\infty}\|x_k^i - x_k^j\| = 0, \forall i, j = 1, \ldots, M$. Furthermore, if it reaches consensus, and $\lim_{k\to\infty}\|x_k^i\|$ approaches to a finite constant, $\forall i = 1, \ldots, M$, then the MAS is said to reach convergent consensus.

Two necessary conditions are assumed (Ma & Zhang, 2010; Ren & Beard, 2005; You & Xie, 2011).

Assumption 1. The pair (A, B) is controllable and the graph $\mathcal{G}$ contains a spanning tree.

The objective of this study is to design RHC-based consensus strategies for the system in (1) with communication topology $\mathcal{G}$ subject to the control constraint, and further investigate under what conditions, the designed RHC strategies are distributed, feasible and can achieve consensus and/or convergent consensus.

3. Set stability and inverse optimality

3.1. Preliminary results for set stability

Consider a discrete-time system

$$x_{k+1} = f(x_k),\ k \in \mathbb{Z}_+, \tag{3}$$

where $x_k \in \mathbb{R}^n$, and $f : \mathbb{R}^n \to \mathbb{R}^n$, is continuous. The solution to (3) is denoted by $x(k, x_0)$ with the initial state x_0. Let $\mathcal{O}$ a nonempty closed subset of $\mathbb{R}^n$, and $\mathcal{O}$ is not necessarily compact. The set $\mathcal{O}$ is said to be forward invariant for the system in (3), if for any $x_0 \in \mathcal{O}$, it follows that $x(k, x_0) \in \mathcal{O}$, for any $k \geqslant 0$.

Motivated by the set stability definition in Jiang and Wang (2002), we present the definition of asymptotic stability as follows.

Definition 2. For the system in (3), suppose that there is a forward invariant set $\mathcal{O}$. It is said to be asymptotically stable with respect to the set $\mathcal{O}$, if the following two conditions hold: (1) Lyapunov stability: for every $\epsilon > 0$, there exists some $\delta > 0$ such that $|x_0|_{\mathcal{O}} < \delta \Rightarrow |x(k, x_0)|_{\mathcal{O}} < \epsilon, \forall k \geqslant 0$. (2) Attraction: for $x_0 \in \mathcal{X} \subseteq \mathbb{R}^n$, $\lim_{k\to\infty}|x(k, x_0)|_{\mathcal{O}} = 0$.

Theorem 3 (*Jiang & Wang, 2002*). *For the system in (3) with a given forward invariant set $\mathcal{O} \in \mathbb{R}^n$, if there exists a continuous function $V : \mathbb{R}^n \to \mathbb{R}_{\geqslant 0}$, such that (1) $\alpha_1(|x|_{\mathcal{O}}) \leqslant V(x) \leqslant \alpha_2(|x|_{\mathcal{O}})$; (2) $V(f(x)) - V(x) \leqslant -\alpha_3(|x|_{\mathcal{O}})$, for any $x \in \mathcal{X} \subseteq \mathbb{R}^n$, where α_1 and α_2 are $\mathcal{K}$-function, and α_3 is a positive function, then the system in (3) is asymptotically stable with respect to the set $\mathcal{O}$.*

According to Definition 1, the MAS in (1) achieves consensus, meaning that the state for each agent will eventually converge to the consensus set $\mathcal{C} \triangleq \{x^1 = x^2 = \cdots = x^M\}$. By Definition 2, the asymptotic stability with respect to the set $\mathcal{C}$ for the closed-loop system in (2) ensures the state X_k will enter the set $\mathcal{C}$ when $k \to \infty$, implying that the MAS in (1) reaches consensus. As a result, we can show the state consensus by proving that the closed-loop system is asymptotically stable with respect to a consensus set.