Automatica 91 (2018) 36–42

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stability analysis of a class of switched nonlinear systems with delays: A trajectory-based comparison method☆

Xingwen Liu [a], Qianchuan Zhao [b,*], Shouming Zhong [c]

[a] College of Electrical and Information Engineering, Southwest Minzu University, Chengdu, Sichuan, 610041, China
[b] Center for Intelligent & Networked Systems, Department of Automation & TNList Tsinghua University, Beijing, 100084, China
[c] School of Mathematical Sciences, University of Electronic Science & Technology of China, Chengdu, Sichuan, 611731, China

ARTICLE INFO

Article history:
Received 22 January 2017
Received in revised form 5 November 2017
Accepted 5 December 2017

Keywords:
Delays
Nonlinear systems
Positive systems
Stability
Switched systems

ABSTRACT

This paper addresses the stability issue of delayed switched nonlinear systems whose subsystem is of the form $\dot{\boldsymbol{x}}(t) = A(t, \boldsymbol{x}(t), \boldsymbol{x}(t-d(t)))\boldsymbol{x}(t) + B(t, \boldsymbol{x}(t), \boldsymbol{x}(t-d(t)))\boldsymbol{x}(t-d(t))$, that is, the system matrices depend on time and the present and past states. By means of a trajectory-based comparison approach, stability of original switched nonlinear system is transformed into that of a switched positive linear system called a comparison system. The involved delays are piecewise continuous and may be unbounded. The comparison switched system may consist of unstable subsystems. It is shown that if the comparison system is asymptotically or exponentially stable, then the original system is asymptotically or exponentially stable, globally or locally, depending on the domain chosen to form the comparison system.

1. Introduction

Roughly speaking, a switched system consists of a family of dynamical subsystems and a rule, called a switching signal, that determines the switching manner among the subsystems (Zhang, Xie, Zhang, & Wang, 2014). Many dynamic systems can be modeled as switched systems (Goebel, Sanfelice, & Teel, 2009) which exhibit rich dynamics due to the multiple subsystems and various possible switching signals (Liu & Dang, 2011; Zhao, Feng, & Tse, 2010). It is noticed that remarkable achievements were made during the past several decades in the area of switched systems (Deaecto, Souza, & Geromel, 2015; Sun & Wang, 2013; Yu & Zhao, 2016). Because nonlinearities and delays inevitably appear in practical systems and have complicated impacts on system's performance (Li, Tong, & Li, 2015; Wang, Sun, & Wu, 2015), different dynamic properties of switched nonlinear systems with delays have been investigated for a long period (Xu, Liu, & Teo, 2008). Among those properties, stability has received much more attention than the other ones (Liu & Liu, 2016; Niculescu, 2001).

There are many methods for analyzing stability of dynamic systems with delays and the most frequently used ones are Lyapunov–Krasovskii functional and Lyapunov–Razumikhin approaches. Two aspects should not be ignored: (1) These methods require a Lyapunov–Krasovskii functional or Lyapunov–Razumikhin functional which is sometimes hard to construct and is not easy to produce tight (low conservative) stability conditions (Münz, Ebenbauer, Haag, & Allgöwer, 2009; Sun, Liu, Chen, & Rees, 2010). (2) Additional relatively strong constraints are often imposed on delays, for example, in many papers the delays are required to be differentiable, with a small upper bound of their derivative, or even constant (Chen & Zheng, 2010; Huang, Luo, Wei, & Chen, 2015; Kim, Campbell, & Liu, 2008). In order to overcome these disadvantages, we focus particularly on the comparison approach in the present paper.

Recall first some interesting properties of positive systems with delays. A system is said to be positive if its trajectory remains in the positive quadrant whenever its initial condition is nonnegative. It was revealed that the size of delay does not affect system's stability property (Haddad & Chellaboina, 2004). In other words, its stability is completely determined by system matrices. This property keeps true even in the situation of delays being unbounded (Liu & Lam, 2013; Zappavigna, Charalambous, & Knorn, 2012). Moreover, if the size of delay is known, then one can easily estimate the convergent rate of the trajectories of a positive system (Feyzmahdavian, Charalambous, & Johansson, 2014; Shen & Zheng, 2015). Having these properties, positive systems can serve as candidate comparison systems when one studies some dynamic systems where the Lyapunov theory does not work well.

Now review the comparison approach briefly. Stability of a class of delay-free switched nonlinear systems was deduced

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hiroshi Ito under the direction of Editor Daniel Liberzon.

* Corresponding author.
E-mail addresses: xingwenliu@gmail.com, xwliu@uestc.edu.cn (X. Liu), zhaoqc@tsinghua.edu.cn (Q. Zhao), zhongsm@uestc.edu.cn (S. Zhong).

https://doi.org/10.1016/j.automatica.2018.01.018

from a scalar comparison system by using Lyapunov functions method (Chatterjee & Liberzon, 2006). Unlike in Chatterjee & Liberzon (2006), many recently reported papers used the approach by comparing the original system with a positive comparison system of the same dimension. Following this approach, exponential stability condition was established for a class of nonlinear systems with delays (Ngoc, 2015), explicit criteria for exponential stability were presented for linear systems with delays and time-varying parameters (Ngoc & Tinh, 2014), and exponential stability of linear neutral time-varying differential systems with constant delays was discussed in Ngoc and Trinh (2016). Mazenc studied the stability for a general family of delayed linear time-varying neutral systems by a similar way (Mazenc, 2015). In recent years, a wealth of literature focused on the stability issues of switched positive systems with delays (Li & Xiang, 2016; Zhao, Zhang, & Shi, 2013), which inspires us to investigate general switched nonlinear systems by means of comparison with switched positive systems. A sufficient asymptotic stability condition was proposed in Sun (2012) for a class of linear switched systems with unbounded delays. On this ground, we will investigate the stability issue of switched nonlinear systems.

Though the comparison approach has been used in Ngoc (2015), Ngoc and Tinh (2014); Ngoc and Trinh (2016) and Mazenc (2015), the method used in these references cannot be extended to our context since it is well-known that some method cannot be directly extended from non-switched to switched systems. The method in Sun (2012) is also inapplicable here, because all the subsystems of the comparison system (Sun, 2012) need to be asymptotically stable, and such a constraint is removed in this paper. Therefore, new method needs to be explored.

The main contribution of the present paper lies in the following two aspects: (1) A new trajectory-based comparison method is proposed for stability analysis. Roughly speaking, the amplitude of trajectory of original system is entrywise less than that of comparison system provided that the amplitude of initial condition of original system is entrywise less than that of comparison system. (2) It is proved that asymptotic or exponential stability of the comparison system means that of the original system, globally or locally, depending on the considered domain of the original system.

Notation: $|a|$ is the absolute value of a real number a. $\mathbf{0}$ stands for the n-dimensional zero vector. A^{T} is the transpose of matrix A. A is said to be a Metzler matrix if all its off diagonal entries are nonnegative. $A \succeq (\succ, \preceq, \prec)0$ means that all elements of A are nonnegative (positive, nonpositive, negative). $\boldsymbol{\lambda} \succeq (\succ, \preceq, \prec)\mathbf{0}$ can be used in an obvious manner for vector $\boldsymbol{\lambda}$. $\mathbb{R}^{n\times m}$ denotes the set of real $n \times m$-dimensional matrices and $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. $\mathbb{N}_0$ stands for the set of nonnegative integers and $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$. $\underline{m} = \{1, \ldots, m\}$ with $m \in \mathbb{N}$. Given $\boldsymbol{x} = [x_1, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $\|\boldsymbol{x}\|_\infty = \max\{|x_1|, \ldots, |x_n|\}$ is the l_∞ norm $\boldsymbol{x}$, and for simplicity, is denoted by $\|\boldsymbol{x}\|$, and $|\boldsymbol{x}| = [|x_1|, \ldots, |x_n|]^{\mathrm{T}}$. $\vec{\mathcal{B}}_a = \{\boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x}\| \le a\}$. $\mathcal{C}([a,b], X)$ is the set of continuous functions from interval $[a,b]$ to X. For any continuous function $\boldsymbol{x}(s)$ on $[-d, a)$ with scalars $a > 0$, $d > 0$ and any $t \in [0, a)$, $\boldsymbol{x}_t$ denotes a continuous function on $[t-d, t]$ defined by $\boldsymbol{x}_t(\theta) = \boldsymbol{x}(t+\theta)$ for $-d \le \theta \le 0$; $\|\boldsymbol{x}_t\| = \sup_{t-d\le s\le t}\{\|\boldsymbol{x}(s)\|\}$.

2. Preliminaries and problem statements

Consider the following system:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A_{\sigma(t)}\left(t, \boldsymbol{x}(t), \boldsymbol{x}\left(t - d_{\sigma(t)}(t)\right)\right)\boldsymbol{x}(t) + \\ &B_{\sigma(t)}\left(t, \boldsymbol{x}(t), \boldsymbol{x}\left(t - d_{\sigma(t)}(t)\right)\right)\boldsymbol{x}\left(t - d_{\sigma(t)}(t)\right), t \ge t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), t \in [t_0 - d, t_0]\end{aligned} \quad (1)$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state variable, the mapping $\sigma : [t_0, \infty) \to \underline{m}$ is a switching signal with m being the number of subsystems, which is piecewise constant and continuous from the right. For each $l \in \underline{m}$, $A_l(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d_l(t)))$, $B_l(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d_l(t))) \in \mathbb{R}^{n\times n}$ are system matrices which depend on t, the present state $\boldsymbol{x}(t)$, and the delayed state $\boldsymbol{x}(t - d_l(t))$, and therefore each subsystem in (1) is generally a nonlinear system; delay $d_l(t)$ is assumed to be piecewise continuous in t and satisfies

$$0 \le d_l(t), t_0 - d_l \le t - d_l(t), \lim_{t\to\infty}(t - d_l(t)) = \infty \quad (2)$$

with d_l being nonnegative constants, $d = \max_{l\in\underline{m}}\{d_l\}$. $\boldsymbol{\varphi} \in \mathcal{C}([t_0 - d, t_0], \mathbb{R}^n)$ is an initial vector-valued function. In system (1), the switching signal σ has finitely many switchings on any finite interval, which means that σ may have finitely many discontinuities (the switching sequence is $\{t_i\}_{i=0}^q$ with $q \in \mathbb{N}$ and $t_i > t_{i-1}, \forall i \in \underline{q}$) or infinitely many discontinuities (switching sequence is $\{t_i\}_{i=0}^\infty$ satisfying $t_i > t_{i-1} (\forall i \in \mathbb{N})$ and $t_i \to \infty$ as $i \to \infty$). It is assumed that the switching signals are minimal, that is, $\sigma(t_i) \ne \sigma(t_{i-1})$, $i \in \underline{q}$ or $i \in \mathbb{N}$.

Let us consider the following non-switched system:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= A(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d(t)))\boldsymbol{x}(t) + \\ &\quad B(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d(t)))\boldsymbol{x}(t - d(t)), \quad t \ge t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), \quad t \in [t_0 - d, t_0]\end{aligned} \quad (3)$$

where $d(t)$ satisfies

$$0 \le d(t), t_0 - d \le t - d(t), \quad \lim_{t\to\infty}(t - d(t)) = \infty. \quad (4)$$

If (3) is a linear time-invariant system, then $A(t, \boldsymbol{x}, \boldsymbol{y}) = A$, $B(t, \boldsymbol{x}, \boldsymbol{y}) = B$. If (3) is a linear time-varying system, then $A(t, \boldsymbol{x}, \boldsymbol{y}) = A(t)$, $B(t, \boldsymbol{x}, \boldsymbol{y}) = B(t)$. For homogeneous system of degree one (Feyzmahdavian et al., 2014), $A(t, \boldsymbol{x}, \boldsymbol{y}) = A(\boldsymbol{x})$, $B(t, \boldsymbol{x}, \boldsymbol{y}) = B(\boldsymbol{y})$ satisfying $A(\lambda\boldsymbol{x}) = A(\boldsymbol{x})$, $B(\lambda\boldsymbol{x}) = B(\boldsymbol{x})$, $\forall\lambda > 0$. For system $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d(t)))$, where $\boldsymbol{f}$ has first partial derivative in the second and third arguments and $\boldsymbol{f}(t, \mathbf{0}, \mathbf{0}) = \mathbf{0}$, one has $\boldsymbol{f}(t, \boldsymbol{x}, \boldsymbol{y}) = \int_0^1 \frac{\partial \boldsymbol{f}(t, s\boldsymbol{x}, s\boldsymbol{y})}{\partial(s\boldsymbol{x})}\boldsymbol{x}\mathrm{d}s + \int_0^1 \frac{\partial \boldsymbol{f}(t, s\boldsymbol{x}, s\boldsymbol{y})}{\partial(s\boldsymbol{y})}\boldsymbol{y}\mathrm{d}s$. Thus, system $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{x}(t - d(t)))$ can be rewritten as (3) with $A(t, \boldsymbol{x}, \boldsymbol{y}) = \int_0^1 \frac{\partial \boldsymbol{f}(t, s\boldsymbol{x}, s\boldsymbol{y})}{\partial(s\boldsymbol{x})}\mathrm{d}s$, $B(t, \boldsymbol{x}, \boldsymbol{y}) = \int_0^1 \frac{\partial \boldsymbol{f}(t, s\boldsymbol{x}, s\boldsymbol{y})}{\partial(s\boldsymbol{y})}\mathrm{d}s$. Therefore, many switched nonlinear systems can be modeled by (1).

Denote $A(t, \boldsymbol{x}, \boldsymbol{y}) = \left[a_{ij}(t, \cdot)\right]$, $B(t, \boldsymbol{x}, \boldsymbol{y}) = \left[b_{ij}(t, \cdot)\right]$, $A_l(t, \boldsymbol{x}, \boldsymbol{y}) = \left[a_{ij}^{(l)}(t, \cdot)\right]$, $B_l(t, \boldsymbol{x}, \boldsymbol{y}) = \left[b_{ij}^{(l)}(t, \cdot)\right]$. Let $D = \vec{\mathcal{B}}_r$ or $D = \mathbb{R}^n$. The next assumption is required:

Assumption 1. There exist real numbers $a_{ij}, b_{ij} (i, j \in \underline{n})$, $a_{ij}^{(l)}$, $b_{ij}^{(l)} (i, j \in \underline{n}, l \in \underline{m})$ such that

$$a_{ij} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D}\left|a_{ij}(t, \cdot)\right|, i \ne j,$$

$$a_{ij}^{(l)} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D}\left|a_{ij}^{(l)}(t, \cdot)\right|, i \ne j,$$

$$a_{ii} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D} a_{ii}(t, \cdot), a_{ii}^{(l)} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D} a_{ii}^{(l)}(t, \cdot),$$

$$b_{ij} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D}\left|b_{ij}(t, \cdot)\right|, b_{ij}^{(l)} = \sup_{t\ge t_0, \boldsymbol{x}, \boldsymbol{y}\in D}\left|b_{ij}^{(l)}(t, \cdot)\right|.$$

Define

$$A = \left[a_{ij}\right], B = \left[b_{ij}\right], A_l = \left[a_{ij}^{(l)}\right], B_l = \left[b_{ij}^{(l)}\right]. \quad (5)$$

Clearly, A, A_l are Metzler matrices and B, B_l are nonnegative matrices.

Introduce the comparison systems of (1) and (3)

$$\begin{aligned}\dot{\boldsymbol{y}}(t) &= A_{\sigma(t)}\boldsymbol{y}(t) + B_{\sigma(t)}\boldsymbol{y}\left(t - d_{\sigma(t)}(t)\right), \quad t \ge t_0 \\ \boldsymbol{y}(t) &= \boldsymbol{\phi}(t), \quad t \in [t_0 - d, t_0]\end{aligned} \quad (6)$$