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Brief paper

Distributed output regulation of heterogeneous linear multi-agent systems with communication constraints[☆]

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ABSTRACT

This paper deals with the cooperative output regulation problem for heterogeneous linear multi-agent systems in the presence of communication constraints. We present a distributed control algorithm, relying on mild assumptions on the directed graph topology, that solves the problem with intermittent and asynchronous discrete-time information exchange and unknown time-varying delays and possible information losses. A numerical example is given to illustrate our results.

1. Introduction

Distributed coordination in multi-agent systems has recently gained extensive attention due to its potential applications in engineering, biological and social systems (Ren & Cao, 2011). The main goal in distributed coordinated control is to realize a group objective using local interaction between agents. From this perspective, various coordinated control algorithms for identical linear multi-agent systems have been proposed in the literature under some assumptions on the interconnection topology. Examples of these results can be found in Abdessameud and Tayebi (2010, 2013), Cao, Ren, and Egerstedt (2012), Li, Duan, Chen, and Huang (2010), Li, Wen, Duan, and Ren (2015) and Scardovi and Sepulchre (2009), where several methods have been proposed to solve different, yet closely-related, coordination problems including consensus, cooperative tracking, and synchronization. For heterogeneous multi-agent systems, Wieland, Sepulchre, and Allgöwer (2011) have shown that distributed algorithms can also be derived using similar methods combined with results from the classical output regulation theory (Francis & Wonham, 1976). In fact, the cooperative output regulation problem has emerged as an important problem that encapsulates many coordinated control problems of heterogeneous agents.

In this paper, we consider the cooperative output regulation problem of heterogeneous linear multi-agent systems governed by the general dynamics

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i \upsilon \\ y_i &= C_i x_i + D_i u_i + F_i \upsilon \end{aligned}, \quad i \in \mathcal{N} := \{1, \ldots, n\}, \tag{1}$$

where $x_i \in \mathbb{R}^{N_{x_i}}$, $u_i \in \mathbb{R}^{N_{u_i}}$, $y_i \in \mathbb{R}^{N_{y_i}}$ are, respectively, the state vector, the control input, and the measured output of the ith agent, and matrices $A_i, B_i, C_i, D_i, E_i, F_i$ are of appropriate dimensions. The signal $\upsilon \in \mathbb{R}^q$ models both the global reference signal to be tracked and the disturbance to be rejected by each agent and is generated by the following exogenous dynamic system

$$\dot{\upsilon} = S\upsilon, \tag{2}$$

with $S \in \mathbb{R}^{q\times q}$. The objective consists in designing the input u_i such that stabilization of some regulated error signal, to be defined later, is guaranteed.

Clearly, in the case where all agents can sense/estimate the exogenous signal υ, the above described problem reduces to the output regulation problem of a single plant studied in Huang (2004). In this work, we are interested in the case where the exogenous signal υ can be estimated only by a group of agents (referred to as leaders) using their output signals, whereas the other agents (referred to as follower) attempt to achieve the control objective by coordinating with other team members. To that end, all agents

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are interconnected in the sense that some information can be transmitted between agents according to some graph topology. Note that the external signal in (2) affects the dynamics of all agents as in (1); The exogenous system is not considered here as an agent that directly transmits its state υ to some agents in the group.

The above problem, with its variants regarding the system model, has been addressed in Liu, De Persis, and Cao (2015), Meng, Yang, Dimarogonas, and Johansson (2015), Su and Huang (2012a, 2012b) and Xiang, Wei, and Li (2009), to cite a few, under different assumptions on the interconnection graph, however, under similar idealized assumptions on the interaction between agents which is generally performed using communication networks. In fact, in all the above mentioned papers, the information exchange is assumed ideal in the sense that the information is continuously transmitted between agents and received in real time. In practice, however, communication over networks is subject to time-varying delays, packets dropouts, and can be discrete-time and intermittent due to various environmental and/or technological factors. Motivated by this, our main interest in this paper is to solve the cooperative output regulation problem for system (1) assuming constrained discrete-time communication between agents.

The second-order consensus problem for double integrators has been studied, for instance, in Cepeda-Gomez and Olgac (2011), Liu, Xie, and Wang (2014), Sun and Wang (2009) and Yu, Chen, and Cao (2010), in the presence of uniform constant communication delays. For identical higher-order linear multi-agents, Zhou and Lin (2014) presented a consensus algorithm in the presence of arbitrary large constant communication delays. A similar result was also obtained in Zhou and Lin (2014) in the case of uniform time-varying delays, however, under some conditions on the delays upper bounds and some restrictions on the dynamics of the agents. A common assumption in the above mentioned delay-robust algorithms is that the information exchange is assumed to be continuous in time. In Xiang, Li, and Ma (2014), a consensus algorithm for high-order heterogeneous agents has been proposed assuming sampled-data information exchange subject to constant communication delays.

In Gao and Wang (2010), Huang, Duan, and Zhao (2014), Wen, Duan, Ren, and Chen (2013), Wen, Duan, Yu, and Chen (2012) and Zhou, Zhang, Xiang, and Wu (2012), consensus algorithms for linear homogeneous multi-agent systems have been presented in the case of intermittent information exchange between agents. However, communication delays have been considered only in Huang et al. (2014) dealing with the second-order consensus problem under the assumptions of strong connectivity, periodic intermittent communication, and perfectly known time-varying communication delays. More recently, a small-gain framework has been adopted in Abdessameud, Polushin, and Tayebi (2014) to design distributed algorithms for nonlinear second-order systems in the presence of irregular communication delays. The latter approach has been further developed in Abdessameud, Polushin, and Tayebi (2015, 2017) to solve similar problems for second-order systems assuming delayed and (not necessarily periodic) intermittent discrete-time information exchange.

The main contribution of this paper is a solution to the cooperative output regulation problem for heterogeneous linear multi-agent systems assuming discrete-time, intermittent and asynchronous information exchange, subject to non-uniform and unknown irregular communication delays that can be unbounded. As compared to the relevant literature mentioned above, the present work considers the coordinated control problem of high-order heterogeneous multi-agent systems by taking into account all the above communication constraints simultaneously. Our control objective can be reached in the case of a general directed interconnection graph for arbitrary properties of the communication process that can induce large communication blackouts.

2. Problem formulation

2.1. Model description

Consider the n heterogeneous agents in (1) and let $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$ denote the directed graph that models the interconnection between agents, where $\mathcal{N}$ is the set of nodes representing the agents, $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of pairs of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix. An edge $(j, i) \in \mathcal{E}$, represented by a directed link from node j to node i, indicates that agent i can receive information from agent j but not *vice versa*. A finite sequence of distinct edges of $\mathcal{G}$ in the form $(j, l_1), (l_1, l_2), \ldots, (l_p, i)$ is called a directed path from j to i. The elements of $\mathcal{A}$ are defined such that $a_{ii} := 0$, $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. The Laplacian matrix $\mathbf{L} := [l_{ij}] \in \mathbb{R}^{n\times n}$ associated to $\mathcal{G}$ is defined such that: $l_{ii} = \sum_{j=1}^{n} a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

Also, the information exchange is discrete in time and subject to irregular communication delays. More precisely, for each $(j, i) \in \mathcal{E}$, agent j can send data to agent i only at instants $t_{k_{ij}} = k_{ij} t_s$, with $k_{ij} \in \mathcal{S}_{ij} \subseteq \{0, 1, 2, \ldots\}$ and t_s being a common sampling period. This information can be received by agent i at instant $t_{k_{ij}} + \tau_{k_{ij}}$, where $\left(\tau_{k_{ij}}\right)_{k_{ij}\in\mathcal{S}_{ij}}$ is a sequence of communication delays that take values in $\mathbb{R}_+ \cup \{+\infty\}$, where $\tau_{k_{ij}} = +\infty$ means that the corresponding data has been lost during transmission.

2.2. Problem statement and assumptions

Our objective consists in designing a control algorithm for (1)–(2) such that the regulated error signal $e_i \in \mathbb{R}^{p_{e_i}}$ written in the general form

$$e_i = C_{e_i} x_i + D_{e_i} u_i + F_{e_i} \upsilon, \quad i \in \mathcal{N}, \tag{3}$$

satisfies $e_i(t) \to 0$, for $i \in \mathcal{N}$, for all initial conditions.

Let $\mathcal{F} := \{1, \ldots, m\} \subset \mathcal{N}$, with $0 < m < n$, $\mathcal{L} := \mathcal{N} \setminus \mathcal{F}$, and define the matrices[1]

$$\bar{A}_i := \begin{bmatrix} A_i & E_i \\ 0_{q\times N_{x_i}} & S \end{bmatrix}, \quad \bar{C}_i := [C_i \quad F_i], \quad i \in \mathcal{N}. \tag{4}$$

Assumption 1. The eigenvalues of S in (2) lie on the imaginary axis of the complex plane.

Assumption 2.

(i) For $i \in \mathcal{N}$, (A_i, B_i) is stabilizable.
(ii) For $i \in \mathcal{L}$, $(\bar{C}_i, \bar{A}_i)$, is detectable.
(iii) For $i \in \mathcal{F}$, $(\bar{C}_i, \bar{A}_i)$ is not detectable and (C_i, A_i) is detectable.
(iv) For all $i \in \mathcal{N}$, there exist matrices Π_i, Γ_i such that

$$\begin{aligned} \Pi_i S &= A_i \Pi_i + B_i \Gamma_i + E_i \\ 0 &= C_{ei} \Pi_i + D_{ei} \Gamma_i + F_{ei}. \end{aligned} \tag{5}$$

Assumption 2, item (iv) is standard and necessary for the solvability of the output regulation problem (Huang, 2004). Items (ii) and (iii) distinguish the above defined sets $\mathcal{L}$ and $\mathcal{F}$ and imply that the agents belonging to $\mathcal{L}$, referred to as leaders, can estimate their states as well as the external signal υ using their measured outputs. This is not the case for the agents belonging to $\mathcal{F}$, referred to as followers, where only their states (x_i for $i \in \mathcal{F}$) are assumed to be detectable from their measurements. It should be noted that the trivial case where $(\bar{C}_i, \bar{A}_i)$, $i \in \mathcal{F}$, is detectable does not affect the forthcoming analysis.

[1] Throughout the paper, $0_{q\times p} \in \mathbb{R}^{q\times p}$ denotes the matrix of all zeros, 0_q (respectively, I_q) denotes the zero matrix (respectively, identity matrix) of dimension $q \times q$, and $\lim_{t\to+\infty} x(t) = c$ is denoted for short by $x(t) \to c$.