Automatica 89 (2018) 1–7

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Circular formation of networked dynamic unicycles by a distributed dynamic control law☆

Xiao Yu, Xiang Xu, Lu Liu *, Gang Feng

Department of Mechanical and Biomedical Engineering, City University of Hong Kong, Kowloon, Hong Kong Special Administrative Region

ARTICLE INFO

Article history:
Received 2 August 2016
Received in revised form 30 March 2017
Accepted 30 October 2017

Keywords:
Circular formation
Distributed control
Directed graph
Dynamic unicycles
Multi-agent systems

ABSTRACT

This paper investigates the circular formation control problem of networked dynamic unicycles. Each unicycle uses its local coordinate frame and the topology of the networked unicycles is modeled by a directed graph containing a spanning tree. A distributed dynamic control law is proposed for each unicycle based on the measurement via local sensing and the information of its neighbors via intermittent communication. It is shown that all unicycles can globally converge to the circular motion around a given center which is only known to one unicycle, and can globally converge to a desired spaced formation along the circle. Finally, simulation results of an example verify the effectiveness of the proposed control law.

1. Introduction

Recent decade witnesses the rapid development of distributed formation control (Oh, Park, & Ahn, 2015), and graph theory has been extensively utilized in design and analysis (Anderson, Yu, Hendrickx, et al., 2008; Hendrickx, Anderson, Delvenne, & Blondel, 2007). Significant effort has been devoted to formation control of multiple single-integrators, see Dorfler and Francis (2010), Krick, Broucke, and Francis (2009), Lee and Ahn (2016) and Oh and Ahn (2013) and references therein. However, these results cannot be applied to multiple unicycles due to the nonholonomic constraint. As the unicycle model can be used to describe the simplified model of a mobile wheeled robot (MWR) and an unmanned aerial vehicle (UAV) (Qu, 2009), interest in formation control of unicycles has been growing, and many works focus on circular formation control of multiple kinematic or dynamic unicycles.

For networked unicycles with all-to-all communication, Sepulchre, Paley, and Leonard (2007) presented a comprehensive investigation on the circular formation of unicycles with unit linear velocity. Seyboth, Wu, Qin, Yu, and Allgower (2014) studied the case where unicycles maintain nonidentical constant linear velocities. For the cyclic pursuit problem of multiple unicycles, it was shown in Marshall, Broucke, and Francis (2004, 2006) that local stability of the closed-loop system can be established and the equilibrium corresponds to generalized regular polygons formation. Sinha and Ghose (2007) considered the case where unicycles are moving with different linear velocities. Zheng, Lin, and Yan (2009) proposed a projection-based control law and ensured that the trajectories of unicycles will not diverge. For unicycles in the cyclic pursuit manner, many works focused on the case where the center of the common circle is given and known to all unicycles. Ceccarelli, Di Marco, Garulli, and Giannitrapani (2008) took into account the limited visibility region of onboard sensors, and Summers, Akella, and Mears (2009) addressed the spaced formation along the circle based on the rigidity of graphs. Lan, Yan, and Lin (2010) developed a hybrid control law for a target-enclosing task. Zheng, Liu, and Sun (2015) proposed controllers based on bearing-only measurement. For the case where the center is only known to one unicycle, Yu and Liu (2016) proposed a distributed dynamic control law for ring-networked unicycles. Several works investigated the circular formation of unicycles under a network of more general topology. Sepulchre, Paley, and Leonard (2008) proposed a dynamic control law based on a balanced graph condition. Chen and Zhang (2011, 2013) developed a controller based on a jointly connected condition for unicycles under a proximity graph. Note that all aforementioned approaches were developed for kinematic unicycles. While for dynamic unicycles, El-Hawwary and Maggiore (2013) proposed a hierarchical controller design strategy, such that unicycles achieve a circular formation with a desired spacing.

☆ This work was supported by the Research Grants Council of the Hong Kong Special Administrative Region of China under Project CityU/11213415. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: xiaoyu4-c@my.cityu.edu.hk (X. Yu), xiangxu5-c@my.cityu.edu.hk (X. Xu), luliu45@cityu.edu.hk (L. Liu), megfeng@cityu.edu.hk (G. Feng).

https://doi.org/10.1016/j.automatica.2017.11.021

In this paper, we consider a formation control problem of networked dynamic unicycles with respect to a given center only known to one unicycle. The major challenge is to develop a distributed control law for unicycles of which the network topology is a directed graph, such that the global asymptotic stability of the closed-loop system corresponding to a circular formation with any desired spacing can be established. To overcome it, a dynamic control law with a feasible estimate of the center is proposed, and it requires each unicycle to use both sensing and communication, as in Oh and Ahn (2013).

The contribution of this paper mainly lies in the following four aspects. First, the aforementioned results on circular formation of kinematic unicycles cannot be directly extended to dynamic unicycles. In fact, our proposed control law makes dynamic unicycles reduced to kinematic unicycles, which implies that the control law design can be applied to kinematic unicycles. Second, a directed graph with a spanning tree is a more general assumption than most existing works, for example, a complete graph (Sepulchre et al., 2007; Seyboth et al., 2014), a balanced graph (Sepulchre et al., 2008), a cycle (Marshall et al., 2004, 2006; Summers et al., 2009; Yu & Liu, 2016; Zheng et al., 2009), and a connected undirected graph (El-Hawwary & Maggiore, 2013). Third, the assumption that only one unicycle knows the center is obviously less restrictive than the one that all unicycles know the center (Ceccarelli et al., 2008; Lan et al., 2010; Sepulchre et al., 2007; Summers et al., 2009; Zheng et al., 2015). Some works considered an unspecified center (Chen & Zhang, 2011, 2013; El-Hawwary & Maggiore, 2013; Marshall et al., 2004, 2006; Sepulchre et al., 2007, 2008; Seyboth et al., 2014; Zheng et al., 2009), and those results can be extended to the case where only one unicycle knows a given center, by letting a unicycle orbit around the center as the so-called "stubborn" one in Chen and Zhang (2011). However, the stability of the closed-loop system was established on a linearized system (Marshall et al., 2004, 2006; Zheng et al., 2009) or an approximated system (Chen & Zhang, 2011, 2013). While our proposed control law guarantees the global asymptotic stability of the original closed-loop system. Finally, noting that in the aforementioned works, only El-Hawwary and Maggiore (2013) considered the circular formation with any desired spacing, and made unicycles locally converge to a desired spaced formation when the sensor graph is directed. While with our proposed control law, unicycles can achieve global convergence to any desired spaced formation along the circle.

The rest of this paper is organized as follows. In Section 2, the problem formulation and three technical lemmas are given. Section 3 presents the main results and Section 4 shows the simulation results of an illustrative example. Finally, the conclusion is drawn in Section 5.

Notations: Throughout the paper, $\|\boldsymbol{x}\|$ denotes the 2-norm of a vector $\boldsymbol{x} \in \mathbb{R}^n$.

2. Preliminaries

2.1. Problem formulation

Consider N dynamic unicycles in the form of:

$$\dot{x}_i = v_i \cos\theta_i,\ \dot{y}_i = v_i \sin\theta_i,\ \dot{\theta}_i = \omega_i,$$
$$\dot{v}_i = F_i/I,\ \dot{\omega}_i = T_i/J,\ i = 1, \dots, N, \tag{1}$$

where $\boldsymbol{p}_i := [x_i\ y_i]^{\mathrm{T}} \in \mathbb{R}^2$ is the coordinate of the position and $\theta_i \in \mathbb{R}$ is the heading angle of unicycle i in the inertial frame, $v_i \in \mathbb{R}$ and $\omega_i \in \mathbb{R}$ are the linear velocity and the angular velocity respectively. The control inputs are the torques F_i and T_i, and I and J are constants associated with the moments of inertia.

All unicycles are anonymous and each one only has access to the information of its neighbors in a network. The network topology is described by a directed graph $\mathcal{G} = \{\mathcal{O}, \mathcal{E}\}$ as follows. Digraph $\mathcal{G}$ consists of a finite set of nodes $\mathcal{O} = \{1, \dots, N\}$ representing N unicycles, and a set of edges $\mathcal{E} \subseteq \{(j, i) : j \neq i, i, j \in \mathcal{O}\}$ containing directed edges from node j to node i. A directed edge (j, i) means that the information of unicycle j is available to unicycle i. Denote the Laplacian matrix of $\mathcal{G}$ by matrix $\mathcal{L}$. The following assumption is made on $\mathcal{G}$:

Assumption 1. Digraph $\mathcal{G}$ contains a directed spanning tree with one node, namely node l, being the root. ■

The network is physically set up by the sensors and communication devices of each unicycle. The network topology at an instant t can be further described by a sensor graph $\mathcal{G}_s(t) = \{\mathcal{O}, \mathcal{E}_s(t)\}$ and a communication graph $\mathcal{G}_c(t) = \{\mathcal{O}, \mathcal{E}_c(t)\}$. Define the sets $\mathcal{N}_c^i(t)$ and $\mathcal{N}_s^i(t)$ as $\mathcal{N}_c^i(t) = \{j \in \mathcal{O} | (j, i) \in \mathcal{E}_c(t)\}$ and $\mathcal{N}_s^i(t) = \{j \in \mathcal{O} | (j, i) \in \mathcal{E}_s(t)\}$ respectively. Finally, denote the Laplacian matrices of $\mathcal{G}_s(t)$ and $\mathcal{G}_c(t)$ by $\mathcal{L}_s(t)$ and $\mathcal{L}_c(t)$ respectively. Assume that $\mathcal{G}_s(t)$ is time-invariant as in El-Hawwary and Maggiore (2013) and $\mathcal{G}_c$ is allowed to be time-varying. Then, the following assumption is made.

Assumption 2. There exists an infinite sequence of nonempty, continuous, uniformly bounded and non-overlapping time intervals $[t_n, t_{n+1})$, $n = 0, 1, \dots$, with $t_{n+1} - t_n \le \bar{T}$ for some $\bar{T} > 0$. In each $[t_n, t_{n+1})$, there exists a finite sequence of nonempty and continuous time subintervals $[t_n^k, t_n^{k+1})$, $k = 0, 1, \dots, k_n - 1$, with $t_n^0 = t_n$, $t_n^{k_n} = t_{n+1}$ and $t_n^{k+1} - t_n^k \ge \bar{\tau}$ for $\bar{\tau} > 0$ and an integer k_n. $\mathcal{G}_c(t)$ does not change during each $[t_n^k, t_n^{k+1})$, and the union graph of $\mathcal{G}_c(t)$ during each $[t_n, t_{n+1})$ is $\mathcal{G}$, i.e., $\bigcup_{j=0}^{j=k_n-1} \mathcal{G}_c(t_n^j) = \mathcal{G}$, $n = 0, 1, \dots$. While the sensor graph $\mathcal{G}_s(t)$ satisfies $\mathcal{G}_s(t) = \mathcal{G}$ for all $t \ge t_0$. ■

For the local sensing, when the inertial frame or a common reference direction (Lin, Broucke, & Francis, 2004) is unavailable, the sensors cannot obtain $\boldsymbol{p}_i, \boldsymbol{p}_j$, or $\boldsymbol{p}_i - \boldsymbol{p}_j, j \in \mathcal{N}_i$. In this case, each unicycle can establish its local coordinate frame as in El-Hawwary and Maggiore (2013), i.e., the Frenet–Serret frame. Then, $\boldsymbol{p}_j$, $\boldsymbol{q}_0$, and $\theta_j, j \in \mathcal{N}_i$, measured by unicycle i can be expressed as

$$\boldsymbol{p}_j^i = R(\theta_i)(\boldsymbol{p}_j - \boldsymbol{p}_i),\ \boldsymbol{q}_0^i = R(\theta_i)(\boldsymbol{q}_0 - \boldsymbol{p}_i),\ \theta_j^i = \theta_j - \theta_i,$$

respectively, where $R(\cdot) = \begin{bmatrix} \cos(\cdot) & \sin(\cdot) \\ -\sin(\cdot) & \cos(\cdot) \end{bmatrix}$.

Circular formation control aims at making all unicycles achieve the following objectives: (i) orbiting along a common circle with the center $\boldsymbol{q}_0 := [x_0\ y_0]^{\mathrm{T}}$ and radius r; (ii) maintaining a desired spaced formation described by a vector $\boldsymbol{\alpha} \in \mathbb{R}^N$ along the common circle; (iii) moving with a constant angular velocity ϖ. The vector $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_N]^{\mathrm{T}}$ is used to describe the desired separation angle of two unicycles with respect to the center. $\boldsymbol{\alpha}$ can be set as $\alpha_1 = 0$, $\alpha_N \le 2\pi$, and $\alpha_i \le \alpha_{i+1}$, $i = 1, \dots, N-1$. Then, $\alpha_{ji} := \alpha_j - \alpha_i$ can be viewed as the desired separation angle between unicycles i and j with respect to the center. The *circular formation control problem* in this paper is formally defined as follows.

Problem 1. Consider N networked dynamic unicycles (1). Given a digraph $\mathcal{G}$, a circular formation center $\boldsymbol{q}_0 \in \mathbb{R}^2$, a radius r, and a constant vector $\boldsymbol{\alpha} \in \mathbb{R}^N$ describing the desired spacing, for unicycle $i, i = 1, \dots, N$, with any initial states $[\boldsymbol{p}_i^{\mathrm{T}}(t_0)\ \theta_i(t_0)\ v_i(t_0)\ \omega_i(t_0)]^{\mathrm{T}} \in \mathbb{R}^5$, $\forall t_0 \ge 0$, find a distributed dynamic control law in the form of

$$[F_i\ T_i]^{\mathrm{T}} = \boldsymbol{\sigma}(\boldsymbol{\rho}_i^i, \boldsymbol{p}_j^i, \theta_j^i, \omega_{ji}, r, \alpha_{ji}), \tag{2}$$
$$\dot{\boldsymbol{\rho}}_i^i = \boldsymbol{\kappa}(\boldsymbol{\rho}_i^i, \boldsymbol{\rho}_j^j, \boldsymbol{p}_j^i, \theta_j^i, r, \alpha_{ji}),\ j \in \mathcal{N}_i, \tag{3}$$