Automatica 89 (2018) 103–110

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief Paper

Bilinear Hamiltonian interactions between linear quantum systems via feedback ☆

Symeon Grivopoulos [a,*], Ian R. Petersen [b]

[a] 11 Zafeiriou Rapesi St., Mesolonghi 30200, Greece
[b] Research School of Engineering, The Australian National University, Canberra ACT 2601, Australia

ARTICLE INFO

Article history:
Received 21 November 2016
Received in revised form 6 May 2017
Accepted 12 September 2017

Keywords:
Linear Quantum Stochastic Systems
Field-mediated interactions
Hamiltonian interactions
Coherent quantum control

ABSTRACT

A number of recent works employ bilinear Hamiltonian interactions between Linear Quantum Stochastic Systems (LQSSs). To the authors' knowledge, implementation schemes for such interactions exist only between single-mode systems. In this work, we propose a general method for the implementation of an arbitrary bilinear Hamiltonian interaction between two multi-mode LQSSs via a feedback interconnection. As an application, we show that the direct interaction realization of a certain coherent quantum control architecture is very useful for design and optimization.

1. Introduction

Linear Quantum Stochastic Systems (LQSSs) are a class of models used in quantum optics (Gardiner & Zoller, 2000; Walls & Milburn, 2008; Wiseman & Milburn, 2010), circuit QED systems (Kerckhoff et al., 2013; Matyas, Jirauschek, Peretti, Lugli, & Csaba, 2011), quantum opto-mechanical systems (Dong, Fiore, Kuzyk, & Wang, 2012; Massel et al., 2011; Tsang & Caves, 2010), and elsewhere. The mathematical framework for these models is provided by the theory of quantum Wiener processes, and the associated Quantum Stochastic Differential Equations (Hudson & Parthasarathy, 1984; Meyer, 1995; Parthasarathy, 1999). Potential applications of LQSSs include quantum information processing, quantum measurement and control. In particular, an important application of LQSSs is as coherent quantum feedback controllers for other quantum systems, i.e. controllers that do not perform any measurement on the controlled quantum system, and thus, have the potential to outperform classical controllers, see e.g. Crisafulli, Tezak, Soh, Armen, and Mabuchi (2013), Hamerly and Mabuchi (2012, 2013), James, Nurdin, and Petersen (2008), Maalouf and Petersen (2011), Mabuchi (2008), Nurdin, James, and Petersen (2009), Yanagisawa and Kimura (2003a, b), and Zhang and James (2012).

The ways LQSSs can interact are of particular importance to applications such as the synthesis of larger LQSSs in terms of simple ones, the design of coherent quantum observers and controllers for LQSSs, etc. For concreteness, in this work we shall use terminology and examples from quantum optics. First, the light beam from an output of an LQSS to an input of another LQSS (carrying a quantum optical signal plus quantum noise), provides a directional coupling between the LQSSs. This sort of coupling of LQSSs is referred to as indirect, or *field-mediated interaction*, and, depending on the sort of connection, namely feedforward or feedback, it can be uni- or bidirectional. Additionally, we may have a direct interaction between LQSSs. This occurs when light beams from different LQSSs meet inside an optical device or material (Nurdin, James, & Doherty, 2009). These interactions are always Hamiltonian, hence, bidirectional. In this work, all Hamiltonian interactions between LQSSs are meant to be bilinear, see Section 3. Direct interactions have been considered in Miao, James, and Ugrinovskii (2015), Nurdin, James, and Doherty (2009), Petersen (2014a, b), Petersen and Huntington (2016a), Sichani, Vladimirov, and Petersen (2015) and Zhang and James (2011) for the applications mentioned above.

Contrary to the interactions between physical systems like atoms or elementary particles, that occur naturally, direct interactions between engineered LQSSs must themselves be engineered. In Nurdin, James, and Doherty (2009, Subsection 6.4), a scheme for the implementation of a direct interaction between two single-mode LQSSs (generalized harmonic oscillators), is proposed. In Petersen and Huntington (2015, 2016b), implementations are proposed for the direct coupling observer of Petersen (2014b),

☆ This work was supported by the Australian Research Council under grant number FL110100020, and the Air Force Office of Scientific Research (AFOSR) under agreement number FA2386-16-1-4065. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Oswaldo Luiz V. Costa under the direction of Editor Richard Middleton.

* Corresponding author.
E-mail addresses: symeon.grivopoulos@gmail.com (S. Grivopoulos), ian.petersen@anu.edu.au (I.R. Petersen).

https://doi.org/10.1016/j.automatica.2017.11.002
0005-1098/© 2017 Elsevier Ltd. All rights reserved.

one without, and one with input/output ports for the observer. (In LQSSs, every port is necessarily both an input and an output.) An implementation for the direct coupling observer of Petersen (2014a) is proposed in Petersen and Huntington (2017). All of these implementations are case-specific, however, and involve single-mode LQSSs (and qubits). To the authors' knowledge, there does not exist a scheme in the LQSS literature for the implementation of arbitrary Hamiltonian interactions between arbitrary LQSSs.

It is exactly this issue that the present work addresses. Our scheme entails modifying the original LQSSs by adding extra ports, and creating an isolated feedback loop, by connecting the extra port inputs of one system to the extra port outputs of the other system, and vice-versa. It turns out that, this scheme can realize any bilinear Hamiltonian interaction between the LQSSs. Since the interacting LQSSs can have an arbitrary number of modes and ports, our method does not provide details of the construction to the level that Nurdin, James, and Doherty (2009) and Petersen and Huntington (2015, 2016b, 2017) do, for their specific cases. Nevertheless, the modified LQSSs and the linear static network necessary for the implementation of the direct interaction, see Section 3, can be implemented using the general synthesis results in Braunstein (2005), Leonhardt and Neumaier (2004), Nurdin (2010), Nurdin, James, and Doherty (2009) and Reck, Zeilinger, Bernstein, and Bertani (1994).

We should point out that, for specific LQSSs and desired Hamiltonian interactions, there may exist other, case-specific implementations, such as in Nurdin, James, and Doherty (2009) and Petersen and Huntington (2015, 2016b, 2017), perhaps even optimized in some sense. The proposed implementation, however, is general. Thus, we hope that it will open the door to more extensive use of Hamiltonian interactions in LQSS applications. For example, our method would be useful in implementing coherent quantum controllers employing direct interactions (Miao et al., 2015; Sichani et al., 2015; Zhang & James, 2011), and direct coupling observers for general LQSSs (Petersen & Huntington, 2016a). Furthermore, the equivalence of the feedback architecture considered in this work between two LQSSs, with a Hamiltonian interaction model, can be used to design coherent quantum controllers employing this architecture, as the example of Section 4 demonstrates.

The remainder of the paper is organized, as follows: In Section 2, we establish some notation and terminology used in the paper, and provide a short overview of LQSSs. In Section 3, we introduce the general model of a bilinear Hamiltonian between LQSSs, and present our scheme for its implementation via feedback, see Theorem 1. In Section 4, we present an application to coherent quantum control design. Section 5 concludes the paper.

2. Background material

2.1. Notation and terminology

In this subsection, we establish the notation and terminology used throughout this paper:

(1) x^* denotes the complex conjugate of a complex number x or the adjoint of an operator x, respectively. For a matrix $X = [x_{ij}]$ with complex or operator entries, $X^\# = [x_{ij}^*]$, $X^\top = [x_{ji}]$ is the usual transpose, and $X^\dagger = (X^\#)^\top$. The commutator of two operators X and Y is defined as $[X, Y] = XY - YX$.

(2) The identity matrix in k dimensions will be denoted by I_k, and a $r \times s$ matrix of zeros will be denoted by $0_{r\times s}$. Let $\mathbb{J}_{2k} = \begin{pmatrix} 0_{k\times k} & I_k \\ -I_k & 0_{k\times k}\end{pmatrix}$. When the dimensions can be inferred from context, we shall simply use I, $\mathbf{0}$, and $\mathbb{J}$. δ_{ij} denotes the Kronecker delta symbol, i.e. $I = [\delta_{ij}]$. $\mathrm{diag}(Z_1, Z_2, \ldots, Z_k)$ is the block-diagonal matrix formed by the square matrices $Z_1, Z_2, \ldots, Z_k$, and $\mathrm{Im}A$ denotes the range of a matrix.

(3) For a matrix $X \in \mathbb{C}^{2r\times 2s}$, define its $\sharp$-*adjoint* $X^\sharp$, by $X^\sharp = -\mathbb{J}_{2s}X^\dagger\mathbb{J}_{2r}$. The $\sharp$-*adjoint* satisfies properties similar to the usual adjoint, namely $(x_1A + x_2B)^\sharp = x_1^*A^\sharp + x_2^*B^\sharp$, $(AB)^\sharp = B^\sharp A^\sharp$, and $(A^\sharp)^\sharp = A$.

(4) A matrix $T \in \mathbb{C}^{2k\times 2k}$ is called *symplectic*, if it satisfies $TT^\sharp = T^\sharp T = I_{2k} \Leftrightarrow T\mathbb{J}_{2k}T^\dagger = T^\dagger\mathbb{J}_{2k}T = \mathbb{J}_{2k}$. Hence, any symplectic matrix is invertible, and its inverse is its $\sharp$-adjoint. The set of these matrices forms a non-compact Lie group known as the symplectic group. Real symplectic matrices constitute a subgroup of the (complex) symplectic group.

2.2. Linear quantum stochastic systems

The material in this subsection is fairly standard, and our presentation aims mostly at establishing notation and terminology. To this end, we follow mostly the review paper (Petersen, 2016). For the mathematical background necessary for a precise discussion of LQSSs, some standard references are Hudson & Parthasarathy (1984), Meyer (1995) and Parthasarathy (1999), while for a Physics perspective, see Gardiner & Collett (1985) and Gardiner & Zoller (2000). The references Edwards & Belavkin (2005), Gough, Gohm, & Yanagisawa (2008), Gough & James (2009), Gough, James, & Nurdin (2010) and Nurdin, James, & Doherty (2009) contain a lot of relevant material, as well.

The systems we consider in this work are collections of quantum harmonic oscillators interacting among themselves, as well as with their environment. The ith harmonic oscillator (mode), $i = 1, \ldots, n$, is described by its position and momentum variables, q_i and p_i, respectively. These are self-adjoint operators satisfying the *Canonical Commutation Relations* (CCRs) $[q_i, q_j] = 0$, $[p_i, p_j] = 0$, and $[q_i, p_j] = \imath\delta_{ij}$, for $i, j = 1, \ldots, n$. If we define the vectors of operators $q = (q_1, q_2, \ldots, q_n)^\top$, $p = (p_1, p_2, \ldots, p_n)^\top$, and $x = \binom{q}{p}$, the CCRs can be expressed as

$$[x, x^\top] \doteq xx^\top - (xx^\top)^\top = \begin{pmatrix} \mathbf{0} & \imath I_n \\ -\imath I_n & \mathbf{0}\end{pmatrix} = \imath\mathbb{J}_{2n}. \tag{1}$$

The environment is modelled as a collection of zero temperature bosonic quantum fields. The ith field, $i = 1, \ldots, m$, is described by bosonic *field annihilation and creation operators* $\mathcal{A}_i(t)$ and $\mathcal{A}_i^*(t)$, respectively. The field operators are *adapted quantum stochastic processes* with forward differentials $d\mathcal{A}_i(t) = \mathcal{A}_i(t+dt) - \mathcal{A}_i(t)$, and $d\mathcal{A}_i^*(t) = \mathcal{A}_i^*(t+dt) - \mathcal{A}_i^*(t)$. They satisfy the quantum Itô products $d\mathcal{A}_i(t)d\mathcal{A}_j(t) = 0$, $d\mathcal{A}_i^*(t)d\mathcal{A}_j^*(t) = 0$, $d\mathcal{A}_i^*(t)d\mathcal{A}_j(t) = 0$, and $d\mathcal{A}_i(t)d\mathcal{A}_j^*(t) = \delta_{ij}dt$. If we define the vector of field operators $\mathcal{A}(t) = (\mathcal{A}_1(t), \mathcal{A}_2(t), \ldots, \mathcal{A}_m(t))^\top$, and the vector of self-adjoint *field quadratures*

$$\mathcal{V}(t) = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathcal{A}(t) + \mathcal{A}(t)^\# \\ -\imath(\mathcal{A}(t) - \mathcal{A}(t)^\#)\end{pmatrix},$$

the quantum Itô products above can be expressed as

$$d\mathcal{V}(t)d\mathcal{V}(t)^\top = \frac{1}{2}\begin{pmatrix} I_m & \imath I_m \\ -\imath I_m & I_m\end{pmatrix}dt = \frac{1}{2}(I_{2m} + \imath\mathbb{J}_{2m})dt. \tag{2}$$

To describe the dynamics of the harmonic oscillators and the quantum fields, we introduce certain operators. We begin with the Hamiltonian operator $H = \frac{1}{2}x^\top Rx$, which specifies the dynamics of the harmonic oscillators in the absence of any environmental influence. $R \in \mathbb{R}^{2n\times 2n}$ is a symmetric matrix referred to as the Hamiltonian matrix. Next, we have the coupling operator L (vector of operators) that specifies the interaction of the harmonic oscillators with the quantum fields. L depends linearly on the position and momentum operators of the oscillators, and can be expressed