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A bias-correction method for closed-loop identification of Linear Parameter-Varying systems☆

Manas Mejari [a], Dario Piga [b], Alberto Bemporad [a]

[a] IMT School for Advanced Studies Lucca, Piazza San Francesco 19, 55100 Lucca, Italy
[b] IDSIA Dalle Molle Institute for Artificial Intelligence, SUPSI-USI, 6928 Manno, Switzerland

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ABSTRACT

Due to safety constraints and unstable open-loop dynamics, system identification of many real-world processes often requires gathering data from closed-loop experiments. In this paper, we present a bias-correction scheme for closed-loop identification of *Linear Parameter-Varying Input–Output* (LPV-IO) models, which aims at correcting the bias caused by the correlation between the input signal exciting the process and output noise. The proposed identification algorithm provides a consistent estimate of the open-loop model parameters when both the output signal and the scheduling variable are corrupted by measurement noise. The effectiveness of the proposed methodology is tested in two simulation case studies.

1. Introduction

Many real world systems must be identified based on data collected from closed-loop experiments. This is typical for open-loop unstable plants, where a feedback controller is necessary to perform the experiments, and in many applications in which a controller is needed to keep the system at certain operating points. Safety, performance, and economic requirements are further motivations to operate in closed-loop.

From the system identification point of view, one of the main issues which makes identification from closed-loop experiments more challenging than in the open-loop setting is due to the correlation between the plant input and output noise. If such a correlation is not properly taken into account, approaches that work in open loop may fail when closed-loop data is used (Ljung, 1999). Several remedies have been proposed in the literature to overcome this problem, especially for the *Linear Time-Invariant* (LTI) case (see Forssell & Ljung, 1999; Van den Hof, 1998 for an overview). These approaches can be classified in: *direct methods*, which neglect the existence of the feedback loop and apply prediction error methods directly on the input–output data after properly parametrizing the noise model; *indirect methods*, where the closed-loop system is identified and the model of the open-loop plant is then extracted exploiting the knowledge of the controller and of the feedback structure; *joint input–output methods*, which treat the measured input and output signals as the outputs of an augmented multi-variable system driven by external disturbances. The model of the open-loop process is then extracted based on the estimate of different transfer functions of the augmented system. Unlike indirect methods, an exact knowledge of the controller is not needed.

Unfortunately, the extension of these approaches to the *Linear Parameter-Varying* (LPV) case is not straightforward, mainly because the classical theoretical tools which are commonly used in closed-loop LTI identification no longer hold in the LPV setting (Tóth, 2010), such as transfer functions and commutative properties of operators. Therefore, only few contributions addressing identification of LPV systems from closed-loop data are available in the literature. A subspace method, which can be applied both for open- and closed-loop identification of LPV models, was proposed in van Wingerden and Verhaegen (2009). The idea of this method is to construct a matrix approximating the product between the extended time-varying observability and controllability matrices, and later use an LPV extension of the predictor subspace approach originally proposed in Chiuso (2007). As far as the identification of *LPV Input–Output* (LPV-IO) models is concerned, the closed-loop output error approach proposed in Landau and Karimi (1997) in the LTI setting is extended in Boonto and Werner (2008) to the identification of LPV-IO models, whose parameters are estimated recursively through a parameter adaptation algorithm. *Instrumental-Variable* (IV) based methods are proposed in Abbas

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E-mail addresses: manas.mejari@imtlucca.it (M. Mejari), dario.piga@supsi.ch (D. Piga), alberto.bemporad@imtlucca.it (A. Bemporad).

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and Werner (2009), Ali, Ali, and Werner (2011) and Tóth, Laurain, Gilson, and Garnier (2012). The contribution in Abbas and Werner (2009) is mainly focused on the identification of *quasi*-LPV systems, where the scheduling variable is a function of the output. The main idea in Abbas and Werner (2009) is to recursively estimate the output signal (and thus the scheduling variable) through recursive least-squares and later use the estimated signals (instead of the measurements) to obtain a consistent estimate of the open-loop model parameters through IV methods. An indirect approach is used in Ali et al. (2011), where IV methods are used to estimate a model of the closed-loop system based on pre-filtered external reference and output signals. The plant parameters are later extracted from the estimated closed-loop model using plant-controller separation methods. In Tóth et al. (2012), an iterative *Refined Instrumental Variable* (RIV) approach is proposed for closed-loop identification of LPV-IO models with Box–Jenkins noise structures. At each iteration of the IV algorithm, the signals are pre-filtered by stable LTI filters constructed using the parameters estimated at the previous iteration. The filtered signals are then used to build the instruments, which are used to recompute an (improved) estimate of the model parameters. Unlike the methods in Abbas and Werner (2009) and Ali et al. (2011), which are restricted to the case of LTI controllers, the approach in Tóth et al. (2012) can handle both LTI and LPV controllers.

This paper presents a bias-correction approach for closed-loop identification of LPV systems. The main idea underlying bias-correction methods is to eliminate the bias from ordinary *Least Squares* (LS) to obtain a consistent estimate of the model parameters. Bias-correction methods have been used in the past for the identification of LTI systems both in the open-loop (Hong, Söderström, & Zheng, 2007; Zheng, 2002) and closed-loop setting (Gilson & Van den Hof, 2001; Zheng & Feng, 1997), as well as for open-loop identification of nonlinear (Piga & Tóth, 2014) and LPV systems from noisy scheduling variable observations (Piga, Cox, Tóth, & Laurain, 2015). The main idea behind the closed-loop identification algorithm proposed in this paper is to quantify, based on the available measurements, the asymptotic bias due to the correlation between the plant input and the measurement noise. Recursive relations are derived to compute the asymptotic bias based on the knowledge of the controller and of the closed-loop structure of the system. Furthermore, in order to handle the more realistic scenario where not only the output signal, but also the scheduling variables are corrupted by a measurement noise, the proposed approach is combined with the ideas presented in Piga et al. (2015), with the following improvements:

- an analytic expression, in terms of Hermite polynomials, is provided to compute the bias-correcting term used to handle the noise on the scheduling variable;
- as the bias-correcting term depends on the variance of the noise corrupting the scheduling variable, a bias-corrected cost function is introduced. This cost function serves as a tuning criterion to determine the value of the unknown noise variance via cross-validation.

Overall, the proposed closed-loop LPV identification approach offers a computationally low-demanding algorithm which: (i) provides a consistent estimate of the model parameters; (ii) can be applied under LTI or LPV controller structures; (iii) does not require to identify the closed-loop LPV system; (iv) can handle noisy observations of the scheduling signal.

The paper is organized as follows. The notation used throughout the paper is introduced in Section 2. The considered identification problem is formulated in Section 3. Section 4 describes the proposed closed-loop bias-correction approach that is extended in Section 5 to handle the case of identification from noisy measurements of the scheduling signal. Two case studies are reported in Section 6 to show the effectiveness of the presented method.

2. Notation

Let $\mathbb{R}^n$ be the set of real vectors of dimension n. The ith element of a vector $x \in \mathbb{R}^n$ is denoted by x_i and $\|x\|^2 = x^\top x$ denotes the square of the 2-norm of x. For matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, the Kronecker product between A and B is denoted by $A \otimes B \in \mathbb{R}^{mp\times nq}$. Given a matrix A, the symbol $[A]_{n\times m}$ means that A is a matrix of dimension $n \times m$. Let $\mathbb{I}_a^b$ be the sequence of successive integers $\{a, a+1, \ldots, b\}$, with $a < b$. The floor function is denoted by $\lfloor\cdot\rfloor$, where $\lfloor m \rfloor$ is the largest integer less than or equal to m. The expected value of a function f w.r.t. the random vector $x \in \mathbb{R}^n$ is denoted by $\mathbb{E}_{x_1,\ldots,x_n}\{f(x)\}$. The subscript $x_1, \ldots, x_n$ is dropped from $\mathbb{E}_{x_1,\ldots,x_n}$ when its meaning is clear from the context.

3. Problem formulation

3.1. Data generating system

By referring to Fig. 1, consider the LPV data-generating closed-loop system $\mathcal{S}_\mathrm{o}$. We assume that the plant $\mathcal{G}_\mathrm{o}$ is described by the LPV difference equations with output-error noise

$$\mathcal{G}_\mathrm{o} : \begin{cases} \mathcal{A}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))x(k) = \mathcal{B}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))u(k), \\ y(k) = x(k) + e(k), \end{cases} \tag{1}$$

and that the controller $\mathcal{K}_\mathrm{o}$ is a *known* LPV or LTI system described by

$$\mathcal{K}_\mathrm{o} : \mathcal{C}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))u(k) = \mathcal{D}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))\,(r(k) - y(k))\,, \tag{2}$$

where $r(k)$ is a bounded reference signal of the closed-loop system $\mathcal{S}_\mathrm{o}$; $u(k) \in \mathbb{R}$ and $y(k) \in \mathbb{R}$ are the measured input and output signals of the plant $\mathcal{G}_\mathrm{o}$, respectively; $x(k)$ is noise-free output; $e(k) \sim \mathcal{N}(0, \sigma_e^2)$ is an additive zero-mean white Gaussian noise with variance σ_e^2 corrupting the output signal; $p_\mathrm{o}(k) : \mathbb{N} \to \mathbb{P}$ is the measured (noise-free) scheduling signal and $\mathbb{P} \subseteq \mathbb{R}^{n_\mathrm{p}}$ is a compact set where $p_\mathrm{o}(k)$ is assumed to take values. In order not to make the notation too complex, from now on we assume that $p_\mathrm{o}(k)$ is scalar (i.e., $n_\mathrm{p} = 1$). The operator q denotes the time shift (i.e., $q^{-i}x(k) = x(k-i)$), and $\mathcal{A}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))$, $\mathcal{B}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))$, $\mathcal{C}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))$ and $\mathcal{D}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k))$ are polynomials in q^{-1} of degree n_a, n_b, n_c and $n_d - 1$, respectively, defined as follows:

$$\mathcal{A}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k)) = 1 + \sum_{i=1}^{n_a} a_i^\mathrm{o}(p_\mathrm{o}(k))q^{-i},$$

$$\mathcal{B}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k)) = \sum_{i=1}^{n_b} b_i^\mathrm{o}(p_\mathrm{o}(k))q^{-i},$$

$$\mathcal{C}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k)) = 1 + \sum_{i=1}^{n_c} c_i^\mathrm{o}(p_\mathrm{o}(k))q^{-i},$$

$$\mathcal{D}_\mathrm{o}(q^{-1}, p_\mathrm{o}(k)) = \sum_{i=0}^{n_d-1} d_{i+1}^\mathrm{o}(p_\mathrm{o}(k))q^{-i},$$

where the coefficient functions $a_i^\mathrm{o}, b_i^\mathrm{o}, c_i^\mathrm{o}, d_i^\mathrm{o}$ are supposed to be polynomials in $p_\mathrm{o}(k)$, i.e.,

$$a_i^\mathrm{o}(p_\mathrm{o}(k)) = \bar{a}_{i,0}^\mathrm{o} + \sum_{s=1}^{n_g} \bar{a}_{i,s}^\mathrm{o} p_\mathrm{o}^s(k), \tag{3a}$$

$$b_i^\mathrm{o}(p_\mathrm{o}(k)) = \bar{b}_{i,0}^\mathrm{o} + \sum_{s=1}^{n_g} \bar{b}_{i,s}^\mathrm{o} p_\mathrm{o}^s(k), \tag{3b}$$

$$c_i^\mathrm{o}(p_\mathrm{o}(k)) = \bar{c}_{i,0}^\mathrm{o} + \sum_{s=1}^{n_g} \bar{c}_{i,s}^\mathrm{o} p_\mathrm{o}^s(k), \tag{3c}$$